# A Uniform Continuum Model for Scaling of Ad Hoc Networks

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**Abstract.** This paper models an ad-hoc network as a continuum of nodes, ignoring edge effects, to find how the traffic scales with $N$, the number of nodes. We obtain expressions for the traffic due to application data, packet forwarding, mobility and routing, and we find the effects of the transmission range, $R$, and the bandwidth. The results indicate that the design of scalable adhoc networks should target small numbers of nodes (not over 1000) and short transmission ranges. The analysis produces three dimensionless parameters that characterize the nodes and the network: $\alpha$, the *walk/talk ratio*, or the ratio of the link event rate to the application packet rate; $\beta$, the *forwarding overhead*, or the average number of hops required for a packet to travel from source to destination; and $\gamma$, the *routing overhead*. We find that the quantity $\alpha\gamma/\beta$ characterizes the relative importance of routing traffic and user data traffic. These quantities may be useful to compare the results of various simulation studies.

**Keywords:** Ad-hoc networks, mobile, scaling, continuum, model.

## 1 Introduction

Several features distinguish ad-hoc networks [5] from their traditional wired counterparts: (1) Ad-hoc networks consist of mobile nodes that communicate by relatively low-powered radio signals. (2) Nodes act both as hosts for application software and as routers to forward incoming packets to other nodes. (3) Ad-hoc networks need to be highly dynamic: the nodes should be able to move, including entering and leaving the network, without manual configuration. (4) Finally, since nodes rely on batteries, power is a scarce resource.

Some recent papers have explored algorithms and protocols for this combination of constraints. Santivanez et al. [11] model the scaling of ad-hoc routing protocols. Gupta and Kumar [6] analyze the capacity of wireless networks under a sophisticated model, although mobility and routing are not considered. Hong, Xu and Gerla [7] analyze the scalability and operational features of routing protocols for mobile ad-hoc networks. They divide routing protocols into three categories: flat routing, hierarchical routing and geographical (GPS-augmented) routing.

This paper investigates the scaling of a very simple model with minimum *a priori* assumptions; the effects of mobility and (flat) routing are included. To this end, we construct an ad-hoc network model by specifying just the average density of nodes (the number of nodes per unit area): we are not concerned with discrete nodes, their

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exact positions or movements, or their exact links with other nodes. Thus, our model focuses on a *continuum* approximation rather than a graph-theoretic view of the network.

In order to simplify the analysis, we assume that all nodes see similar traffic conditions. In other words, we ignore the *edge effects* resulting from nodes near the extremity of a network having fewer neighbors than centrally located nodes. In this sense our model is *uniform*.

Our goal is to model the traffic in an ad-hoc network using simple and optimistic assumptions. Our simple assumptions lead to a mathematically tractable model, which in turn reveals several dimensionless parameters that characterize the operation of an ad-hoc network; these may be helpful in bridging the gaps between the parameters of various simulations. Our optimistic assumptions lead to upper bounds on the performance of real networks. We avoid an exclusively asymptotic analysis for a large number of nodes [11] in order to deal with practical finite cases; therefore our results are derived in $cf$ rather than $\Theta(f)$ format, although the values of constants are approximate at best.

In Sections 2 and 3 we define the parameters to describe two-dimensional network geometry and node behavior, respectively. An expression for the traffic due to user applications is derived in Section 4. In Section 5 we find how much traffic results from mobility to support routing. The user data traffic and routing traffic are combined in Section 6 to find the total traffic and the power requirement. The two-dimensional results are extended to $m$ dimensions in Section 7, and conclusions are drawn in Section 8.

## 2 Network Geometry in Two Dimensions

We consider an ad-hoc network whose nodes lie in a plane. With each node we can associate a Voronoi cell, which is the set of points closer to that node than any other. We approximate cells by circles with radii $r_1$ on average, giving an average distance $d_1 = 2r_1$ between neighboring nodes, and we suppose that the cell area is approximately $\pi r_1^2$. (See [6], who investigate feasible Voronoi tessellations in detail.) Thus the *node density* (the number of nodes per unit area) is approximately $4/\pi d_1^2$.

We assume that the network consists of $N$ nodes occupying a circular region of diameter $D+d_1$, and that the maximum distance between any pair of nodes is $D$. The node density must be approximately $4N/(\pi(D+d_1)^2)$, so that $Nd_1^2 = (D+d_1)^2$, or $D = d_1(\sqrt{N}-1)$.

## 3 Node Model

The essential features of nodes are that (a) they generate user data, (b) they forward packets, (c) they move, (d) they have a finite radio transmission range, and (e) they have a finite transmission bandwidth.

(a) We assume that each node is a random source of user data, being characterized by a rate $p_T$, the number of new data packets created and transmitted by a node per unit time to a randomly chosen destination node. (The subscript $T$ is meant to suggest an application transmitting, or *talking*.).

(b) Packet forwarding is discussed in Section 4.

(c) When a node moves, it may enter or leave the radio range of one or more other nodes. We assume that nodes are able to detect the making and breaking of radio links, and that such *link events* occur at each node at a rate $p_W$ per unit time. The symbol $W$ is meant to suggest a user *walking* while carrying a mobile device.

We define $\alpha = p_W/p_T$ to be the *walk/talk ratio*, the ratio of a node's rate of link events to its rate of producing user data packets. Notice that $\alpha$ depends only on the nodal behavior and not on the network configuration. The value $\alpha$ is a useful dimensionless measure of the impact of node mobility.

(d) It is important to model the radio transmission *range* of a node because of a fundamental tradeoff in ad-hoc networks: a large range can reduce the number of hops required to transport a packet to its destination, but it also reduces the number of nodes that can transmit simultaneously. We assume that the transmission range of a node is $R$: at distances exceeding $R$, a node's signal cannot be received and does not interfere with other reception, either because the signal is too weak or because some aspect of the physical layer restricts the nodes' participation. (For example, a frequency hopping scheme may form a logical small-scale network of this size.) We assume that $d_1 \le R \le D$, since (i) for $R < d_1$ the network becomes largely disconnected, and (ii) for $R > D$ all $N$ nodes are already within range. The number of nodes within range of any transmitting node is a group of approximately $g = (R+r_1)^2/r_1^2$ nodes, including the transmitting node.

(e) The quantities $p_T$ and $p_W$ cannot be arbitrarily large because the packet transmission rate for each node is finite. Let $b$ be the maximum possible value for $p_T$, realized when a node transmits new user data packets continuously and handles no other traffic. (Assuming one packet per link event, $p_W$ must also satisfy $p_W \le b$.) Then $b$ is node's *bandwidth* expressed in packets/second. However, because of the nature of a typical physical layer, a node cannot attain a packet transmission rate of $b$. If we assume that the $g$ nodes within radio range share a channel at any moment, the average maximum packet transmission rate per node is only $b/g$.

## 4 User Data Traffic

Our network is assumed to have only two types of traffic: application (*user*) data packets and routing packets. We assume that there is no gateway to other networks. We begin by using the above network geometry and node model to find the traffic due to user data alone.

First we define $\beta$ (>1), the *forwarding overhead*, to be the average number of hops required for a packet to travel from source node to destination node. (We ignore data traffic between local applications.) With the assumption of uniformity, this implies that for every user data packet injected into the network by a node, the node must perform on average $\beta$ packet transmissions. This is a cost of participating in an ad-hoc network: in order to originate data packets at a rate $p_T$, a node must transmit data packets at a rate of $\beta p_T$.

The average distance from the source to the destination [4] is about $D/2$. We assume that routing is optimal, i.e. a packet travels roughly in a straight line in hops of length $R$. Then the distance $D/2$ can be covered in $\beta = D/(2R)$ hops. Since $D = d_1(\sqrt{N}-1)$, we have

$$\beta = \frac{r_1}{R}(\sqrt{N}-1)\,. \tag{1}$$

We note that β is dimensionless, and, because of the limits on $R$, satisfies $1 \le \beta \le (\sqrt{N}-1)/2$. (Following the algebra strictly, $R \le D$ would give $0.5 \le \beta$, but it is precisely as $R$ approaches $D$ that the edge effect begins to matter, and $\beta<1$ makes no sense.) In our model, the forwarding overhead β depends only on the network configuration and not on the nodal behavior. In a more realistic model, it would depend on the routing effectiveness and on traffic patterns.

The overhead β is $\Theta(\sqrt{N}/R)$, which appears to favor large $R$ (but see below). Although dependence on $N$ is not strong, it may still impose restrictions on the number of nodes. For instance, if the overhead is to be no greater than 5 (e.g. for reasons of power consumption), and if the range is as short as possible, $N$ must not exceed 121.

The user data packet rate $\beta p_T$ cannot exceed the limit $b/g$ established earlier: $\beta p_T \le b/g$, or

$$p_T \le \frac{b r_1 R}{(R+r_1)^2}\frac{1}{\sqrt{N}-1}\,. \tag{2}$$

This result shows that small values of $R$ (that is, $R \approx 2r_1$) are preferable. In that case we obtain

$$p_T \le \frac{2b}{9(\sqrt{N}-1)}\,. \tag{3}$$

The limiting case $p_T = \beta b/N$ is obtained for a low walk/talk ratio, where routing traffic can be ignored because the nodes are nearly stationary. For example, if $N$=100 nodes have a bandwidth of 1Mbps each and a range $R = 2r_1$, then each node can transmit only 2Mbps/(9×9) ≈ 25 Kbps on average (including bandwidth overheads such as packet headers), and then only if all nodes are stationary. Throughput drops further if the range increases beyond the minimum of $2r_1$.

We emphasize that these results are independent of any mobility and any routing algorithm. In fact, they would apply to wired networks if hosts were required to perform routing. We note that the finite *minimum traffic load* of Santivanez et al. [11] must already include the effect that $p_T \to 0$ as $N \to \infty$.

## 5 Routing Traffic

Link events are generated by each node at a rate $p_W$. We assume non-hierarchical (flat) proactive routing, that is, a link event is promptly sent as one routing packet to all nodes in order to keep routing tables updated.

We define γ (>1), the *routing overhead*, to be the number of packets generated by one link event. As with user data traffic, uniformity implies that every node must transmit on average γ routing packets per link event, or $\gamma p_W$ packets per unit time. To estimate $\gamma$, we assume that by transmitting one packet a node can broadcast to $g-1$ other nodes. However, the information will be new to only about half of those nodes. In total we must reach $N$ nodes, so that

$$N = \gamma \frac{g-1}{2}, \tag{4}$$

or

$$\gamma = \frac{2N}{\frac{R^2}{r_1^2} + \frac{2R}{r_1}}. \tag{5}$$

If the range is minimized (that is, $R \approx 2r_1$), the routing overhead is $\gamma = (N/4) - 1$.

Clearly this estimate needs to be refined to take specific routing protocols into account. The assumption of reaching $(g-1)/2$ nodes with a single packet may be quite optimistic, since it requires a receiving node to "know" whether its position justifies rebroadcasting a given packet (e.g. whether it is at the edge of the previous transmitter's range in the "forward" direction). On the other hand, some protocols [1,2,9,10] effectively combine multiple events into a single packet to improve efficiency.

The routing overhead is $\Theta(N/R^2)$, and is potentially much more serious than the forwarding overhead, which is only $\Theta(\sqrt{N}/R)$. It is interesting that $\gamma$ is $\Theta(\beta^2)$.

The routing packet rate $\gamma p_W$ cannot exceed the limit $b/g$ established earlier: $\gamma p_W \le b/g$, or

$$p_W \le \frac{b}{\gamma g} = \frac{b}{2N}\frac{g-1}{g} = \frac{b}{2N}\left[1 - \frac{r_1^2}{(R+r_1)^2}\right] \approx \frac{b}{2N}. \tag{6}$$

Therefore the maximum rate at which a node can generate link events is $\Theta(1/N)$. It is almost independent of $R$ because, in our model, $\gamma$ is $\Theta(N/R^2)$ while the bandwidth reduction factor, $1/g$, is $\Theta(R^2)$.

The limiting case $p_W = b/(\gamma g)$ is obtained for a high walk/talk ratio, where the user data traffic is starved to zero by frequent link events. This sets a fundamental limit on the product $Np_W$, the network link event rate. Fortunately, the consequences are not numerically serious for networks of modest size. For example, if $N$=100 nodes have a bandwidth of 1Mbps each, then each node would have to generate almost 5 Kbps of link event packets in order to saturate the network. Similarly, if $p_W$ = 1 event/second and routing packets contain 500 bits, then saturation occurs at about $N$=1000.

# 6 Total Traffic and Power

The combined effect of user data traffic and routing traffic is that each node transmits $\beta p_T + \gamma p_W$ packets per unit time, and the bandwidth constraint for the combined traffic is $\beta p_T + \gamma p_W \leq b/g$. Relative to transmitting $p_T$ packets of user data per unit time, a node incurs an overhead of $\beta + \alpha\gamma$.

Depending on whether $\alpha$, the walk/talk ratio, is greater or less than $\beta/\gamma$, routing traffic or forwarding traffic, respectively, dominates in the network. This suggests using values of $\alpha\gamma/\beta$ in order to compare the results of various simulation studies; see, for example, [1-3,8,10]. In the previous sections, a low (high) walk/talk ratio should be taken to mean $\alpha\gamma/\beta \ll 1$ ($\gg 1$).

Assuming that the range $R$ is limited by a threshold of received signal strength, the average antenna power is proportional to $R^2$ and to the rate of packet transmission. The average power requirement per node is

$$P = k R^2 (\beta p_T + \gamma p_W) \tag{7}$$

where $k$ is a constant. (The antenna power for continuous transmission is $kR^2b$.) The power is $\Theta(N)$, with the leading term arising from mobility. $P$ is the radiated antenna power only, and does not include the power consumption of the node's circuitry itself.

## 7 Other Dimensions

This model has been built for the most practical case of two dimensions, although we could equally well have chosen $m$ dimensions. For $m$=1 we have a model of $N$ nodes spread in a line (e.g. nodes in vehicle on a road), and for $m$=3 we have a model of $N$ nodes spread in a volume (e.g. nodes carried by users in a multi-storied building).

In $m$ dimensions, the node density is measured in units of nodes per (unit length)$^m$. The total number of nodes, $N$, and the network diameter (a distance), $D$, are related by $Nd_1^m = (D+d_1)^m$. The number of nodes in a radio group is $g = (R+r_1)^m / r_1^m$. The data transmission overhead $\beta$ becomes

$$\beta = \frac{r_1}{R}\left(\sqrt[m]{N} - 1\right), \tag{8}$$

the routing overhead $\gamma$ becomes

$$\gamma = \frac{2N}{\left(\frac{R}{r_1}+1\right)^m - 1}. \tag{9}$$

The bandwidth constraints in $m$ dimensions become

$$p_T \leq \frac{b r_1^{m-1} R}{(R+r_1)^m} \frac{1}{\sqrt[m]{N} - 1} \tag{10}$$

and

$$p_W \leq \frac{b}{2N}\left[1 - \frac{r_1^m}{(R+r_1)^m}\right]. \tag{11}$$

For $m$=3, from (10) the maximum $p_T$ is $\Theta(1/R^m N^{1/m})$, which makes it especially important to keep the range small, while $N$ can grow larger than was feasible in two dimensions.

The $R^2$ factor in the power requirement is unchanged because it arises from the three-dimensional spreading of radio signals, regardless of $m$.

# 8 Conclusion

The simple continuum model without edge effects has yielded a number of analytic results. In two dimensions the user data traffic is $\Theta(\sqrt{N}/R)$, and routing traffic is $\Theta(N/R^2)$, where $N$ is the number of nodes and $R$ is the transmission range. The maximum (bandwidth-limited) user data traffic per node is $\Theta(1/R\sqrt{N})$, and the maximum link event rate is $\Theta(1/N)$. It will be interesting to see how closely simulations and real networks follow these scaling trends. Our results confirm that the design of flat ad-hoc networks should target small numbers of nodes (100's, not 1000's), and should strive for short transmission ranges.

This analysis has produced three dimensionless parameters that characterize an ad-hoc network. The node behavior is characterized by $\alpha$, the *walk/talk ratio,* which is the ratio of the link event rate to the application packet rate. The network is characterized by $\beta$, the *forwarding overhead*, and by $\gamma$, the *routing overhead*. We find that the quantity $\alpha\gamma/\beta$ characterizes the relative importance of routing traffic and user data traffic; the two are equal when $\alpha\gamma/\beta = 1$. These dimensionless parameters may prove useful to compare the results of various simulation studies and to scale ad-hoc networks.

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