CONTEXT-FREE GRAMMARS AND SYNTACTIC ANALYSIS

PRINCIPLES OF PROGRAMMING LANGUAGES

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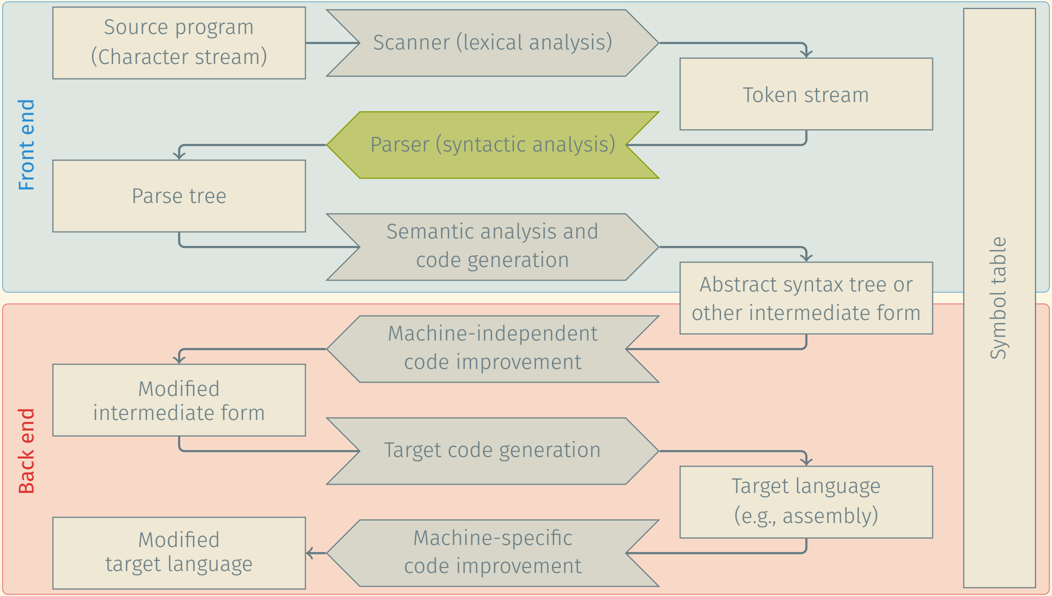
Dalhousie University

PROGRAM TRANSLATION FLOW CHART



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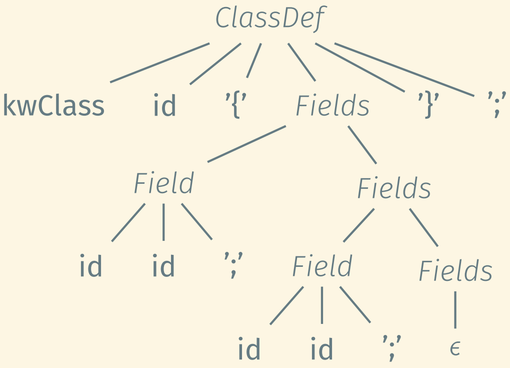

SYNTACTIC ANALYSIS

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Convert the token stream produced by the scanner into a parse tree representing the syntactic structure of the program.

Tools

- Context-free grammars, LL(1)/LR(1) grammars
- (Deterministic) push-down automata, recursive-descent parsers

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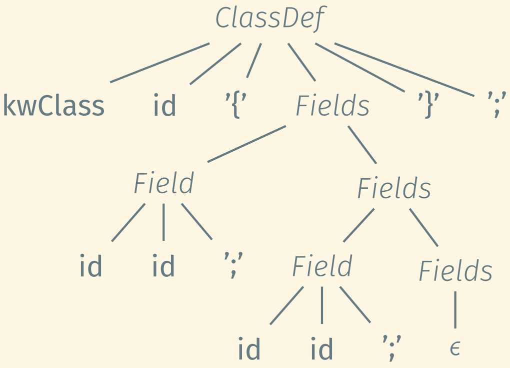

ROAD MAP

- **Parsing:** Transform (tokenized) program text into parse tree
- **Modelling programming languages:** Context-free grammars and languages
- **Capturing the syntactic structure of a program:** Parse trees

- Types of parsers and types of grammars they can parse
- Grammars that describe programming languages and can be parsed efficiently

- Construction of an LL(1) grammar
- Parsing LL(1) languages
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A grammar for a subset of natural language

$$
\begin{aligned}
\textit{Sentence} &\rightarrow \textit{Phrase Verb Phrase}\ \textbf{.} \\
\textit{Phrase} &\rightarrow \textit{Noun} \\
\textit{Phrase} &\rightarrow \textit{Adjective Noun} \\
\textit{Adjective} &\rightarrow \text{'big'} \mid \text{'green'} \\
\textit{Noun} &\rightarrow \text{'cheese'} \mid \text{'Jim'} \\
\textit{Verb} &\rightarrow \text{'ate'}
\end{aligned}
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A grammar for a subset of natural language

$$\mathit{Sentence} \rightarrow \mathit{Phrase}\ \mathit{Verb}\ \mathit{Phrase}\ \textbf{.}$$
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$$\mathit{Verb} \rightarrow \textbf{'ate'}$$

Sentences in the language described by this grammar

- big Jim ate green cheese.
- green Jim ate green cheese.
- Jim ate cheese.
- cheese ate Jim.

A grammar for simple arithmetic expressions

$$Expr \rightarrow \texttt{(}\ Expr\ \texttt{)}$$

$$Expr \rightarrow Expr\ \texttt{+}\ Expr$$

$$Expr \rightarrow \texttt{++}\ Expr$$

$$Expr \rightarrow number$$

$$Expr \rightarrow identifier$$

Definition: Context-free grammar

A quadruple $G = (V, \Sigma, P, S)$ with

- A set of non-terminals or variables V,
- A set of terminals Σ,
- A set of rules or productions in the form

$$V \rightarrow (V \cup \Sigma)^*$$

- A start symbol $S \in V$.

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NOTATIONAL VARIATIONS IN PRODUCTIONS

Merging alternatives using '|':

$Phrase \rightarrow Noun$

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Optional components:

$FunctionDef \rightarrow DeclSpecs_{\mathrm{opt}}\ Decl\ DeclList_{\mathrm{opt}}\ CompoundStmt$

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Regular expression on RHS:

$$Expr \rightarrow Term\ (\text{op}\ Term)^*$$

Definition: Derivation

A sequence of rewriting operations that starts with the string $\sigma = S$ and repeats the following until σ contains only terminals:

- Choose a non-terminal X in σ and a production $X \rightarrow \beta$ in the grammar G.
- Replace X with β in σ.

As a picture:

$$\sigma = \alpha X \gamma \Rightarrow_G \sigma' = \alpha\beta\gamma$$

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DERIVATIONS

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The intermediate strings are called sentential forms.

LANGUAGE DEFINED BY A CONTEXT-FREE GRAMMAR

We write $S \Rightarrow_G^* \sigma$ if there exists a sequence $S \Rightarrow_G \sigma_1 \Rightarrow_G \sigma_2 \Rightarrow_G \cdots \Rightarrow_G \sigma$.

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Example:

The "Jim-ate-cheese" grammar defines the language

$$\mathcal{L}(G) = \{\textbf{Jim ate cheese}, \textbf{Jim ate Jim}, \textbf{big Jim ate cheese}, \ldots\}.$$

CONTEXT-FREE LANGUAGES: EXAMPLES

What is the language defined by the following grammar?

$$S \to \epsilon$$

$$S \to 0\ S\ 0$$

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Neither of these two languages is regular!

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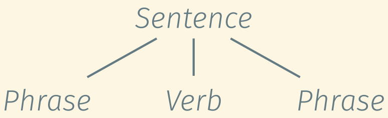

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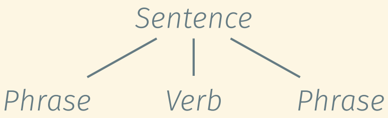

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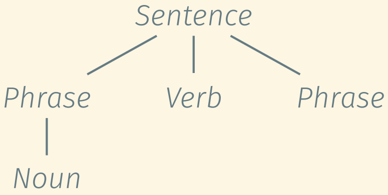

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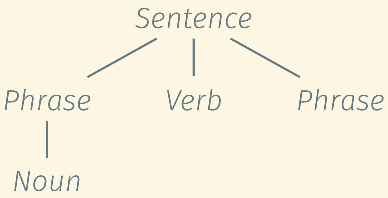

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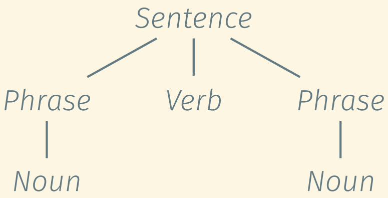

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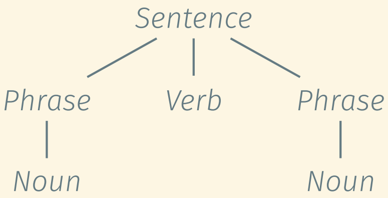

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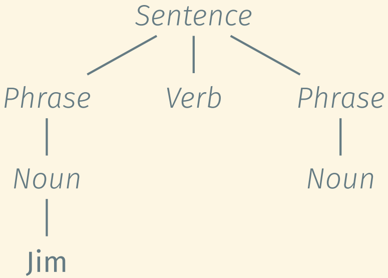

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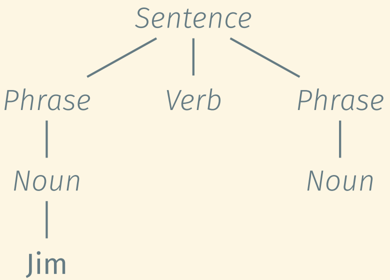

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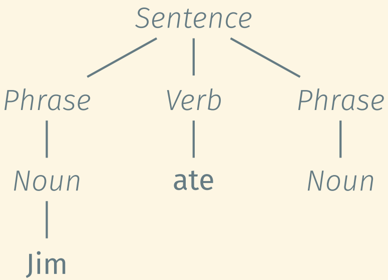
\begin{aligned}
\mathit{Sentence} &\Rightarrow \mathit{Phrase}\ \mathit{Verb}\ \mathit{Phrase} \\
&\Rightarrow \mathit{Noun}\ \mathit{Verb}\ \mathit{Phrase} \\
&\Rightarrow \mathit{Noun}\ \mathit{Verb}\ \mathit{Noun} \\
&\Rightarrow \textbf{Jim}\ \mathit{Verb}\ \mathit{Noun} \\
&\Rightarrow \textbf{Jim ate}\ \mathit{Noun}
\end{aligned}
$$



PARSE TREES

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- The root is *S*.
- The leaves, called the yield of the parse tree, are terminals.
- Every internal node is a non-terminal.
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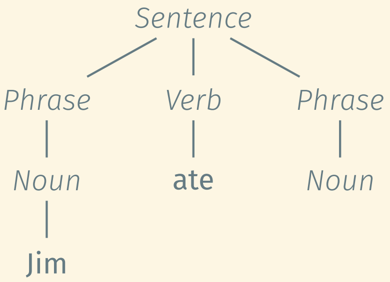
Sentence ⇒ *Phrase Verb Phrase*
⇒ *Noun Verb Phrase*
⇒ *Noun Verb Noun*
⇒ **Jim** *Verb Noun*
⇒ **Jim ate** *Noun*
⇒ **Jim ate cheese**



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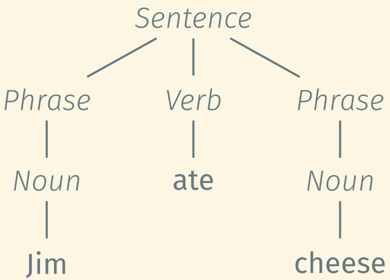
$$\begin{aligned} Sentence &\Rightarrow Phrase\ Verb\ Phrase \\ &\Rightarrow Noun\ Verb\ Phrase \\ &\Rightarrow Noun\ Verb\ Noun \\ &\Rightarrow \textbf{Jim}\ Verb\ Noun \\ &\Rightarrow \textbf{Jim ate}\ Noun \\ &\Rightarrow \textbf{Jim ate cheese} \end{aligned}$$



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⇒ *Noun Verb Phrase*
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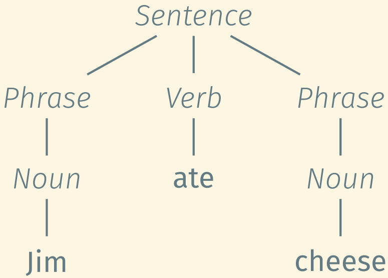
⇒ Jim *Verb Noun*
⇒ Jim ate *Noun*
⇒ Jim ate cheese



Note: In general, there are multiple derivations with the same parse tree.

There are infinitely many context-free grammars generating a given context-free language:

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Some context-free languages are inherently ambiguous, that is, do not have unambiguous grammars. Usually, this is not the case for programming languages.

AMBIGUITY: EXAMPLE (1)

$$2 + 3 * 4$$

$E \rightarrow \text{num}$

$E \rightarrow \text{id}$

$E \rightarrow E \text{ '+' } E$

$E \rightarrow E \text{ '-' } E$

$E \rightarrow E \text{ '*' } E$

$E \rightarrow E \text{ '/' } E$

$E \rightarrow \text{'('} \; E \; \text{')'}$

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$E \rightarrow E$ '/' E

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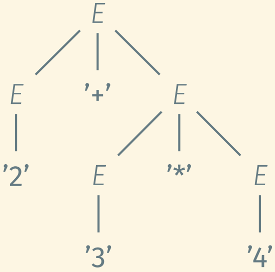

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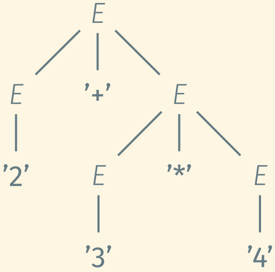

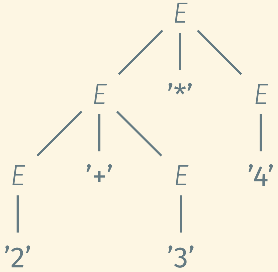

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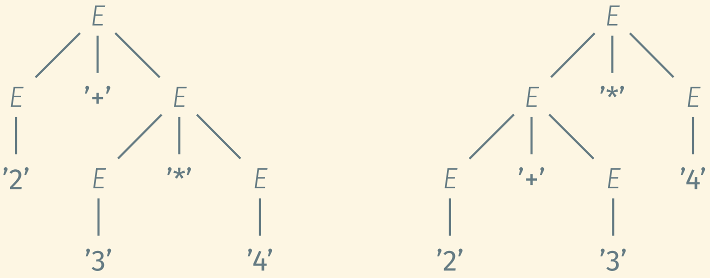

This grammar is ambiguous!

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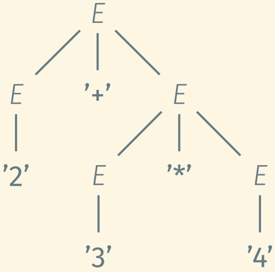

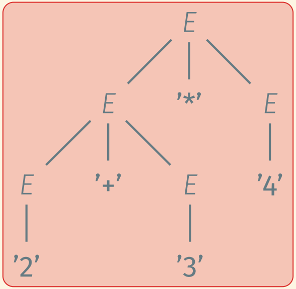

Violates precedence rules!

This grammar is ambiguous!

AMBIGUITY: EXAMPLE (2)

An unambiguous grammar for the same language:

$E \rightarrow T$

$E \rightarrow E$ '+' T

$E \rightarrow E$ '-' T

$T \rightarrow F$

$T \rightarrow T$ '*' F

$T \rightarrow T$ '/' F

$F \rightarrow$ num

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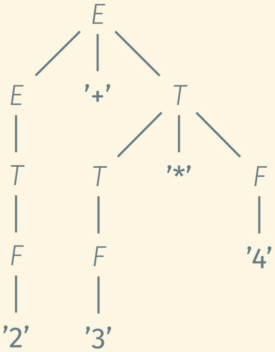

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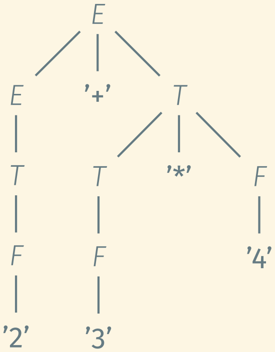

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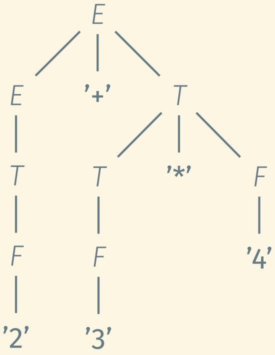

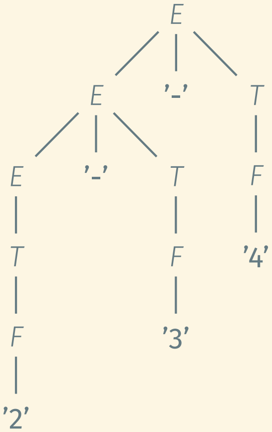

This grammar respects precedence rules.

It also respects left-associativity.

LEFTMOST AND RIGHTMOST DERIVATIONS

For every language defined by an unambiguous grammar, there is only one parse tree that generates each sentence in the language.

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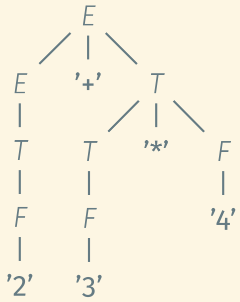

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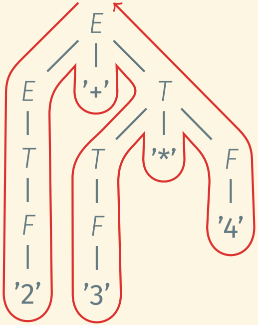
Leftmost derivation: Replaces the leftmost non-terminal in each step.

$$
\begin{aligned}
E &\Rightarrow E \text{ '+' } T \\
&\Rightarrow T \text{ '+' } T \\
&\Rightarrow F \text{ '+' } T \\
&\Rightarrow \text{'2' '+' } T \\
&\Rightarrow \text{'2' '+' } T \text{ '*' } F \\
&\Rightarrow \text{'2' '+' } F \text{ '*' } F \\
&\Rightarrow \text{'2' '+' '3' '*' } F \\
&\Rightarrow \text{'2' '+' '3' '*' '4'}
\end{aligned}
$$



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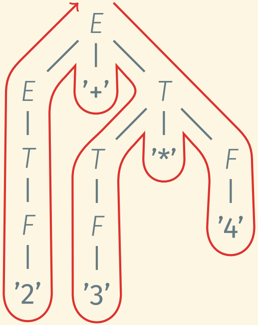
Leftmost derivation: Replaces the leftmost non-terminal in each step.

Rightmost derivation Replaces the rightmost non-terminal in each step.

$$\begin{aligned} E &\Rightarrow E\ \text{'+'}\ T \\ &\Rightarrow T\ \text{'+'}\ T \\ &\Rightarrow F\ \text{'+'}\ T \\ &\Rightarrow \text{'2'}\ \text{'+'}\ T \\ &\Rightarrow \text{'2'}\ \text{'+'}\ T\ \text{'*'}\ F \\ &\Rightarrow \text{'2'}\ \text{'+'}\ F\ \text{'*'}\ F \\ &\Rightarrow \text{'2'}\ \text{'+'}\ \text{'3'}\ \text{'*'}\ F \\ &\Rightarrow \text{'2'}\ \text{'+'}\ \text{'3'}\ \text{'*'}\ \text{'4'} \end{aligned}$$



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- **Modelling programming languages:** Context-free grammars and languages
- **Capturing the syntactic structure of a program:** Parse trees

- Types of parsers and types of grammars they can parse
- Grammars that describe programming languages and can be parsed efficiently

- Construction of an LL(1) grammar
- Parsing LL(1) languages
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EFFICIENT PARSERS

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Most common parser types:

- Recursive-descent parser
- Shift-reduce parser

RECURSIVE-DESCENT PARSING

- Start with the start symbol ($\sigma = S$)

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It is one form of top-down parsing because we start with the start symbol S and construct the parse tree top-down.

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' $S\ S$

$S \rightarrow$ '-' $S\ S$

$S \rightarrow$ '*' $S\ S$

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Parse tree

S_1

Input string	Stack
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+ * + 2 3 4 5	+ S_2 S_3

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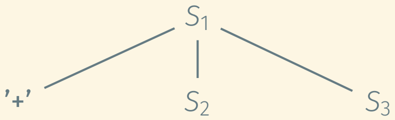

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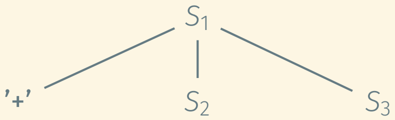

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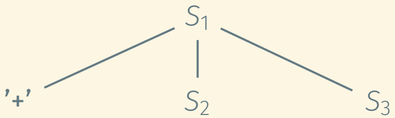

Input string	Stack
* + 2 3 4 5	$S_2\ S_3$

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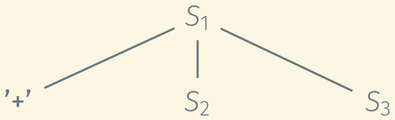

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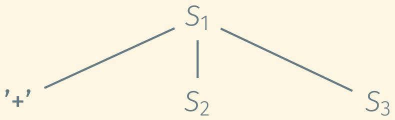

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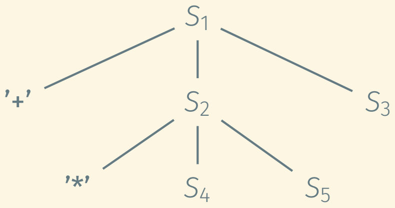

Input string	Stack
* + 2 3 4 5	* S_4 S_5 S_3

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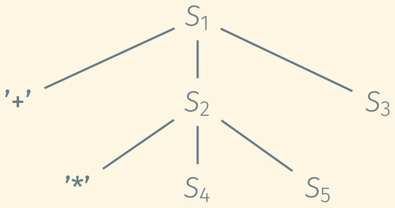
$S \rightarrow$ **'/'** $S\ S$

$S \rightarrow$ **'neg'** S

$S \rightarrow$ **int**

(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
* + 2 3 4 5	* S_4 S_5 S_3

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' $S\ S$

$S \rightarrow$ '-' $S\ S$

$S \rightarrow$ '*' $S\ S$

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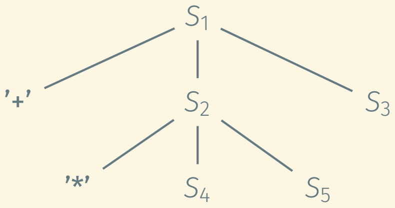
$S \rightarrow$ 'neg' S

$S \rightarrow$ int

(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
+ 2 3 4 5	$S_4\ S_5\ S_3$

RECURSIVE-DESCENT PARSING: EXAMPLE

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' $S\ S$

$S \rightarrow$ '-' $S\ S$

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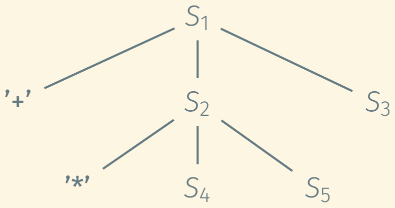
(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
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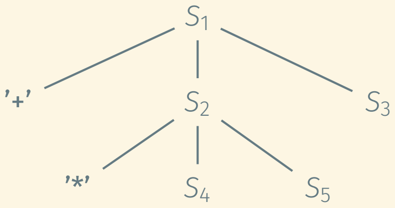
(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
+ 2 3 4 5	+ S_6 S_7 S_5 S_3

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' $S\ S$

$S \rightarrow$ '-' $S\ S$

$S \rightarrow$ '*' $S\ S$

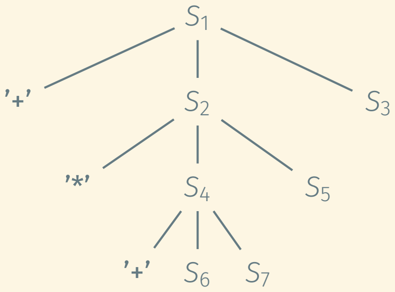
$S \rightarrow$ '/' $S\ S$

$S \rightarrow$ 'neg' S

$S \rightarrow$ int

(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
+ 2 3 4 5	+ S_6 S_7 S_5 S_3

RECURSIVE-DESCENT PARSING: EXAMPLE

An S-grammar for arithmetic expressions in Polish notation:

$$S \rightarrow \text{'+'}\ S\ S$$

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$$S \rightarrow \text{'*'}\ S\ S$$

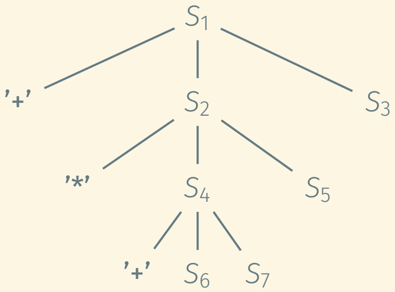
$$S \rightarrow \text{'/'}\ S\ S$$

$$S \rightarrow \text{'neg'}\ S$$

$$S \rightarrow \text{int}$$

$(2 + 3) * 4 + 5 \Rightarrow + * + 2\ 3\ 4\ 5$

Parse tree



Input string	Stack
+ 2 3 4 5	+ S_6 S_7 S_5 S_3

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' S S

$S \rightarrow$ '-' S S

$S \rightarrow$ '*' S S

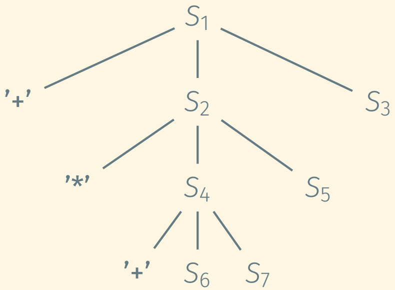
$S \rightarrow$ '/' S S

$S \rightarrow$ 'neg' S

$S \rightarrow$ int

(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
2 3 4 5	S_6 S_7 S_5 S_3

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ **'+'** $S\ S$

$S \rightarrow$ **'-'** $S\ S$

$S \rightarrow$ **'*'** $S\ S$

$S \rightarrow$ **'/'** $S\ S$

$S \rightarrow$ **'neg'** S

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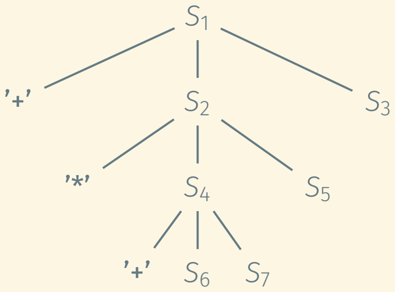
(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
2 3 4 5	$S_6\ S_7\ S_5\ S_3$

RECURSIVE-DESCENT PARSING: EXAMPLE

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' $S\ S$

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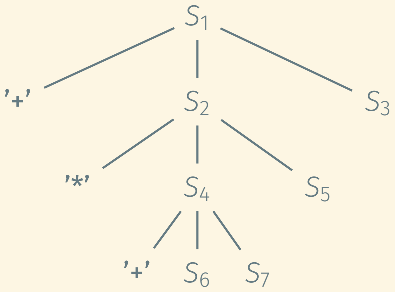

Input string	Stack
2 3 4 5	int S_7 S_5 S_3

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An S-grammar for arithmetic expressions in Polish notation:

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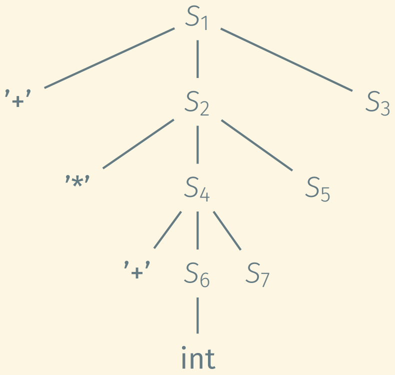
Parse tree



Input string	Stack
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$S \rightarrow$ '+' S S

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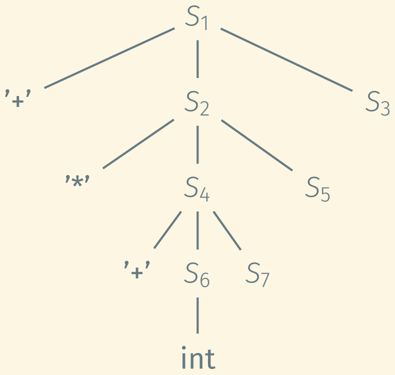

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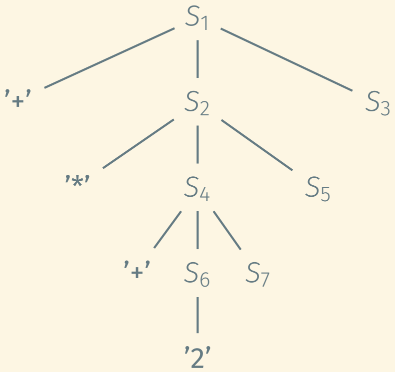
Parse tree



Input string	Stack
3 4 5	S_7 S_5 S_3

RECURSIVE-DESCENT PARSING: EXAMPLE

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ **'+'** $S\ S$

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$S \rightarrow$ **'/'** $S\ S$

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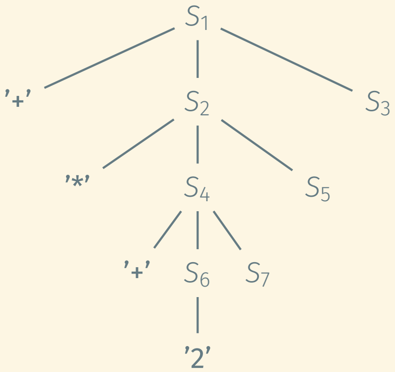
(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
3 4 5	$S_7\ S_5\ S_3$

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' S S

$S \rightarrow$ '-' S S

$S \rightarrow$ '*' S S

$S \rightarrow$ '/' S S

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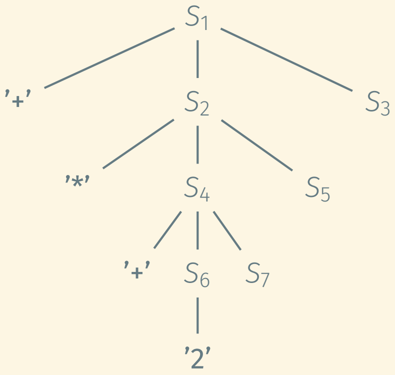
(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
3 4 5	int S_5 S_3

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ **'+'** $S\ S$

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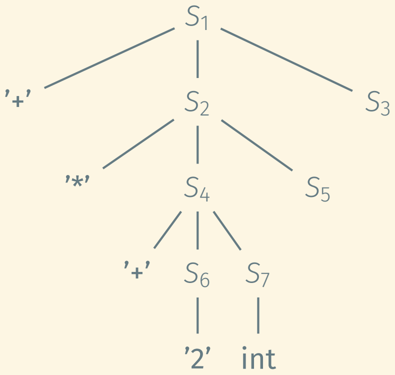
(2 + 3) * 4 + 5 ⇒ + * + 2 3 4 5

Parse tree



Input string	Stack
3 4 5	int S_5 S_3

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' $S\ S$

$S \rightarrow$ '-' $S\ S$

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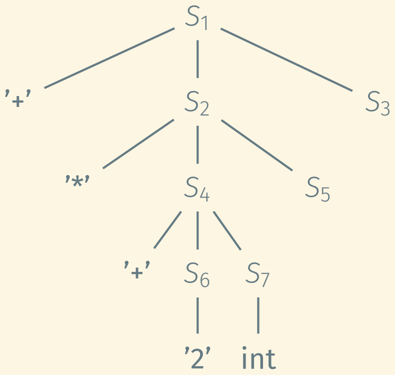
Parse tree



Input string	Stack
3 4 5	int S_5 S_3

An S-grammar for arithmetic expressions in Polish notation:

$S \to$ '+' S S

$S \to$ '-' S S

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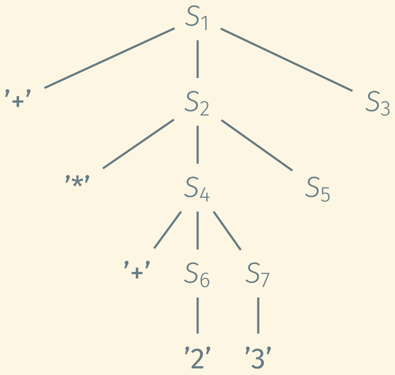
$S \to$ int

(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
4 5	S_5 S_3

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' $S\ S$

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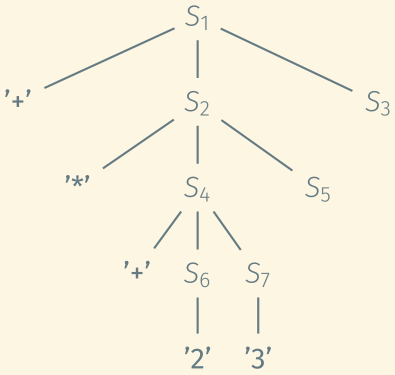
$S \rightarrow$ int

(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
4 5	$S_5\ S_3$

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' S S

$S \rightarrow$ '-' S S

$S \rightarrow$ '*' S S

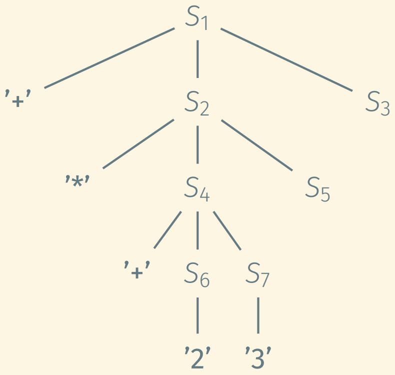
$S \rightarrow$ '/' S S

$S \rightarrow$ 'neg' S

$S \rightarrow$ int

(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
4 5	int S_3

RECURSIVE-DESCENT PARSING: EXAMPLE

An S-grammar for arithmetic expressions in Polish notation:

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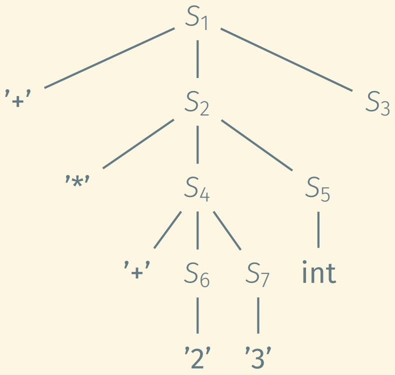
(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
4 5	int S_3

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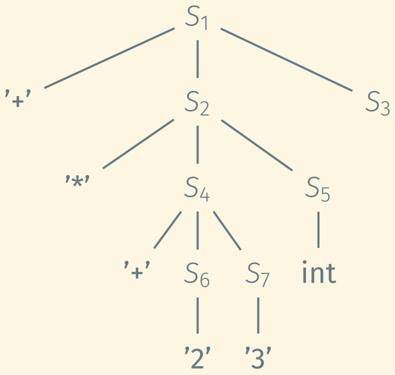
Parse tree



Input string	Stack
4 5	int S_3

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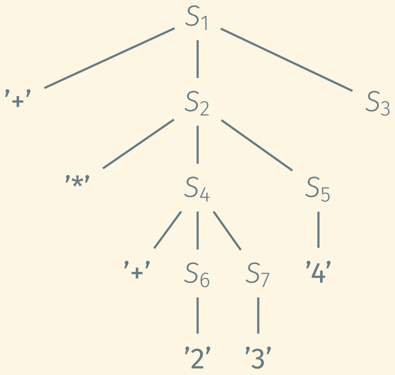

Input string	Stack
5	S_3

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' S S

$S \rightarrow$ '-' S S

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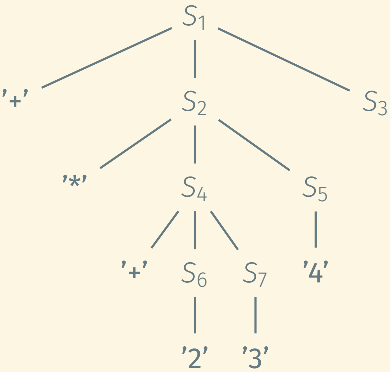
$S \rightarrow$ int

(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
5	S_3

An S-grammar for arithmetic expressions in Polish notation:

$S \to$ '+' $S\ S$

$S \to$ '-' $S\ S$

$S \to$ '*' $S\ S$

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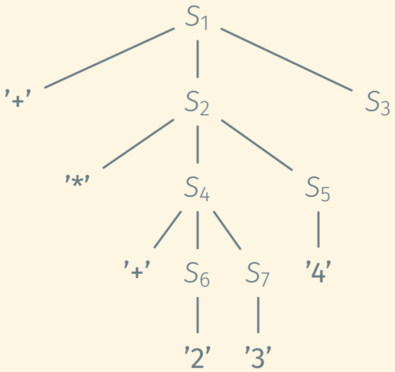
$S \to$ 'neg' S

$S \to$ int

(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
5	int

An S-grammar for arithmetic expressions in Polish notation:

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$S \rightarrow$ '-' S S

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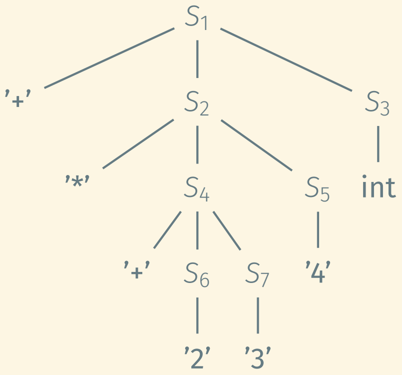
(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
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$S \rightarrow$ '+' $S\ S$

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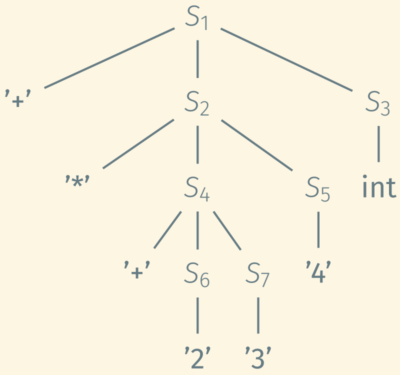
(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string	Stack
5	int

An S-grammar for arithmetic expressions in Polish notation:

$S \rightarrow$ '+' $S\ S$

$S \rightarrow$ '-' $S\ S$

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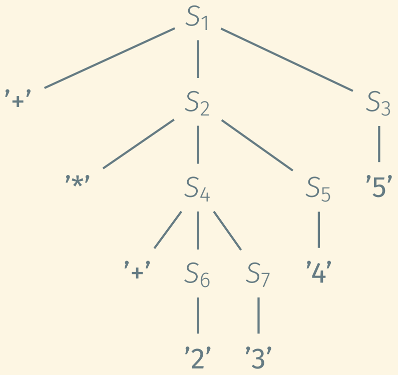
$S \rightarrow$ int

(2 + 3) * 4 + 5 $\Rightarrow$ + * + 2 3 4 5

Parse tree



Input string Stack

- Begin with an empty stack R

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SHIFT-REDUCE PARSING

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  - Shift the next input symbol from τ to R (remove it from τ and push it onto R).

SHIFT-REDUCE PARSING

- Begin with an empty stack R
- While R does not contain the start symbol S and the input $\tau \neq \epsilon$, either
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  - Reduce R by choosing a production $X \to \rho$ such that ρ is the sequence of symbols on the top of R and replace ρ with X in R.

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- While R does not contain the start symbol S and the input $\tau \neq \epsilon$, either
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  - Reduce R by choosing a production $X \to \rho$ such that ρ is the sequence of symbols on the top of R and replace ρ with X in R.
- The parse succeeds if and only if S is the only symbol on R and $\tau = \epsilon$ at the end.

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This is also called LR-parsing because it consumes the input Left-to-right and produce a Rightmost derivation in reverse.

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This is also called LR-parsing because it consumes the input Left-to-right and produce a Rightmost derivation in reverse.

It is a form of bottom-up parsing because it produces the parse tree from the leaves up.

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **’+’**

$S \rightarrow S\ S$ **’-’**

$S \rightarrow S\ S$ **’*’**

$S \rightarrow S\ S$ ’/’

$S \rightarrow S$ ’neg’

$S \rightarrow$ int

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$$S \rightarrow S\ S\ \text{'+'}$$
$$S \rightarrow S\ S\ \text{'-'}$$
$$S \rightarrow S\ S\ \text{'*'}$$
$$S \rightarrow S\ S\ \text{'/'}$$
$$S \rightarrow S\ \text{'neg'}$$
$$S \rightarrow \text{int}$$

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

SHIFT-REDUCE PARSING: EXAMPLE

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$S \rightarrow S\ S$ '+'

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$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 ⇒ 2 3 4 + * 5 +

Parse tree

Input string	Stack
2 3 4 + * 5 +	

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree

Input string	Stack
3 4 + * 5 +	2

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree

'2'

Input string	Stack
3 4 + * 5 +	2

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree

'2'

Input string	Stack
3 4 + * 5 +	2

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

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$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree

S_1

|

'2'

Input string	Stack
3 4 + * 5 +	2

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree

Input string	Stack
3 4 + * 5 +	S_1

A grammar for arithmetic expressions in reverse Polish notation:

$S \to S\ S$ '+'
$S \to S\ S$ '-'
$S \to S\ S$ '*'
$S \to S\ S$ '/'
$S \to S$ 'neg'
$S \to$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree

S_1
|
'2'

Input string	Stack
3 4 + * 5 +	S_1

A grammar for arithmetic expressions in reverse Polish notation:

$$S \rightarrow S\ S\ \text{'+'}$$
$$S \rightarrow S\ S\ \text{'-'}$$
$$S \rightarrow S\ S\ \text{'*'}$$
$$S \rightarrow S\ S\ \text{'/'}$$
$$S \rightarrow S\ \text{'neg'}$$
$$S \rightarrow \text{int}$$

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree

Input string	Stack
4 + * 5 +	S_1 3

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

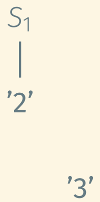
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 ⇒ 2 3 4 + * 5 +

Parse tree



Input string	Stack
4 + * 5 +	S_1 3

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree

S_1
|
'2'

'3'

Input string	Stack
4 + * 5 +	S_1 3

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

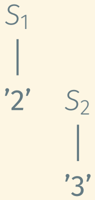
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
4 + * 5 +	S_1 3

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

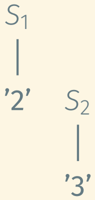
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
4 + * 5 +	$S_1\ S_2$

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

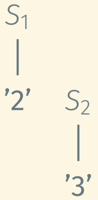
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
4 + * 5 +	$S_1\ S_2$

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

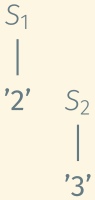
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
+ * 5 +	S_1 S_2 4

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

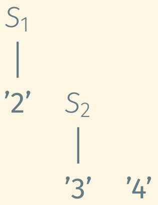
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
+ * 5 +	$S_1\ S_2$ 4

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ ’+’

$S \rightarrow S\ S$ ’-’

$S \rightarrow S\ S$ ’*’

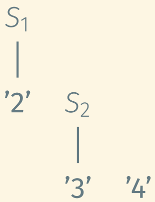
$S \rightarrow S\ S$ ’/’

$S \rightarrow S$ ’neg’

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
+ * 5 +	$S_1\ S_2$ 4

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

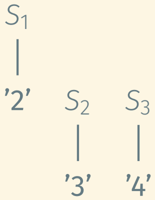
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
+ * 5 +	S_1 S_2 4

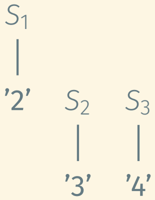
SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'
$S \rightarrow S\ S$ '-'
$S \rightarrow S\ S$ '*'
$S \rightarrow S\ S$ '/'
$S \rightarrow S$ 'neg'
$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
+ * 5 +	$S_1\ S_2\ S_3$

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

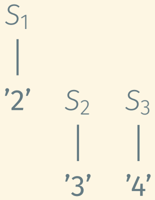
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
+ * 5 +	$S_1\ S_2\ S_3$

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

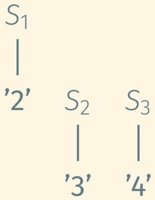
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
* 5 +	$S_1\ S_2\ S_3$ +

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

$S \rightarrow S\ S$ **'-'**

$S \rightarrow S\ S$ **'*'**

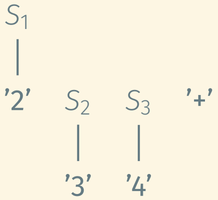
$S \rightarrow S\ S$ **'/'**

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



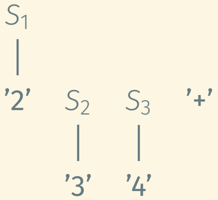
Input string	Stack
* 5 +	S_1 S_2 S_3 +

A grammar for arithmetic expressions in reverse Polish notation:

- $S \rightarrow S\ S$ **'+'**
- $S \rightarrow S\ S$ **'-'**
- $S \rightarrow S\ S$ **'*'**
- $S \rightarrow S\ S$ '/'
- $S \rightarrow S$ 'neg'
- $S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
* 5 +	S_1 S_2 S_3 +

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

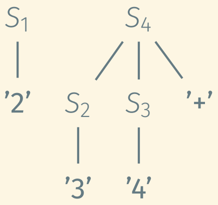
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
* 5 +	$S_1\ S_2\ S_3$ +

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

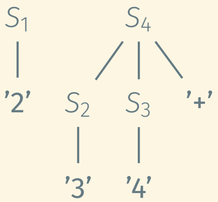
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
* 5 +	$S_1\ S_4$

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

$S \rightarrow S\ S$ **'-'**

$S \rightarrow S\ S$ **'*'**

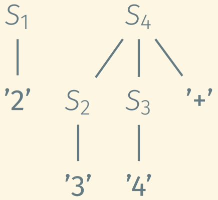
$S \rightarrow S\ S$ **'/'**

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
* 5 +	$S_1\ S_4$

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

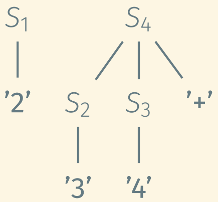
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
5 +	$S_1\ S_4$ *

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

$S \rightarrow S\ S$ **'-'**

$S \rightarrow S\ S$ **'*'**

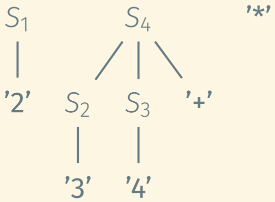
$S \rightarrow S\ S$ **'/'**

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
5 +	S_1 S_4 *

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

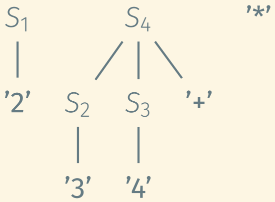
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

$2 * (3 + 4) + 5 \Rightarrow 2\ 3\ 4 + * \ 5 +$

Parse tree



Input string	Stack
5 +	$S_1\ S_4$ *

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

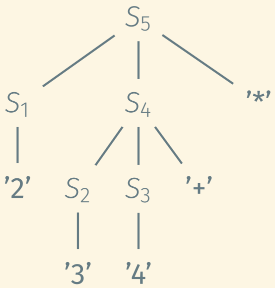
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 ⇒ 2 3 4 + * 5 +

Parse tree



Input string	Stack
5 +	$S_1\ S_4$ *

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

$S \rightarrow S\ S$ **'-'**

$S \rightarrow S\ S$ **'*'**

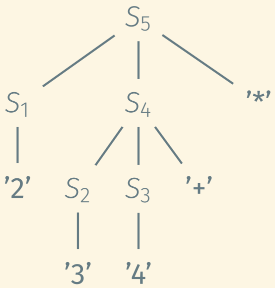
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
5 +	S_5

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \to S\ S$ **'+'**

$S \to S\ S$ **'-'**

$S \to S\ S$ **'*'**

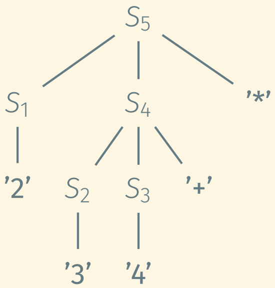
$S \to S\ S$ **'/'**

$S \to S$ **'neg'**

$S \to$ **int**

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
5 +	S_5

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

$S \rightarrow S\ S$ **'-'**

$S \rightarrow S\ S$ **'*'**

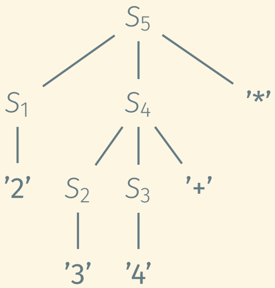
$S \rightarrow S\ S$ **'/'**

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

$2 * (3 + 4) + 5 \Rightarrow 2\ 3\ 4 + * \ 5\ +$

Parse tree



Input string	Stack
+	S_5 **5**

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

$S \rightarrow S\ S$ **'-'**

$S \rightarrow S\ S$ **'*'**

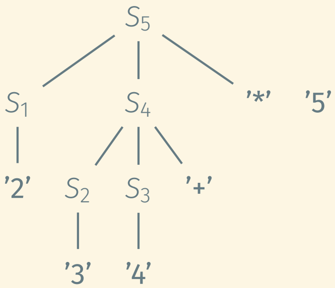
$S \rightarrow S\ S$ **'/'**

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
+	S_5 **5**

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

$S \rightarrow S\ S$ **'-'**

$S \rightarrow S\ S$ **'*'**

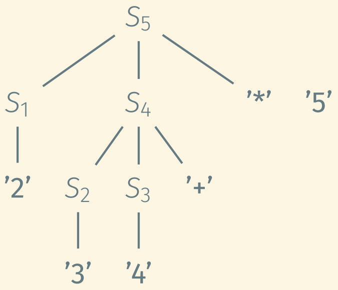
$S \rightarrow S\ S$ **'/'**

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 ⇒ 2 3 4 + * 5 +

Parse tree



Input string	Stack
+	S_5 5

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

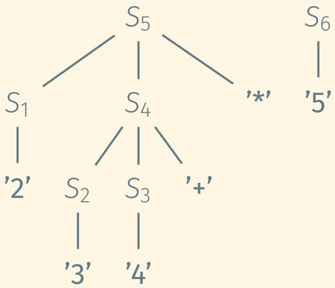
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
+	S_5 5

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

$S \rightarrow S\ S$ **'-'**

$S \rightarrow S\ S$ **'*'**

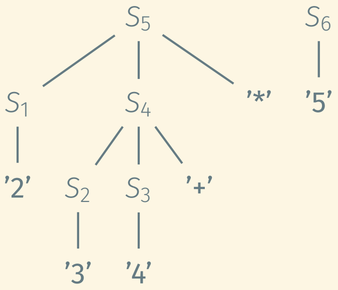
$S \rightarrow S\ S$ **'/'**

$S \rightarrow S$ **'neg'**

$S \rightarrow$ **int**

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
+	$S_5\ S_6$

SHIFT-REDUCE PARSING: EXAMPLE

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

$S \rightarrow S\ S$ **'-'**

$S \rightarrow S\ S$ **'*'**

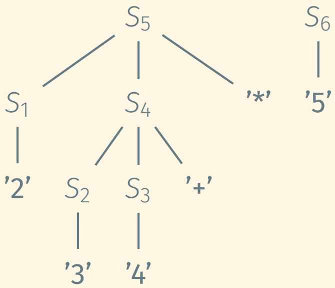
$S \rightarrow S\ S$ **'/'**

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



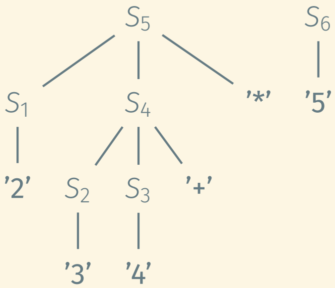
Input string	Stack
+	$S_5\ S_6$

A grammar for arithmetic expressions in reverse Polish notation:

$$S \rightarrow S\ S\ \text{'+'}$$
$$S \rightarrow S\ S\ \text{'-'}$$
$$S \rightarrow S\ S\ \text{'*'}$$
$$S \rightarrow S\ S\ \text{'/'}$$
$$S \rightarrow S\ \text{'neg'}$$
$$S \rightarrow \text{int}$$

$2 * (3 + 4) + 5 \Rightarrow 2\ 3\ 4 + * 5 +$

Parse tree



Input string	Stack
	$S_5\ S_6$ +

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

$S \rightarrow S\ S$ **'-'**

$S \rightarrow S\ S$ **'*'**

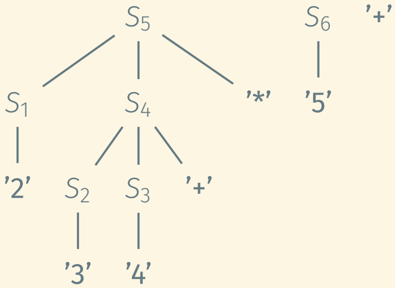
$S \rightarrow S\ S$ **'/'**

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string

Stack

$S_5\ S_6$ +

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

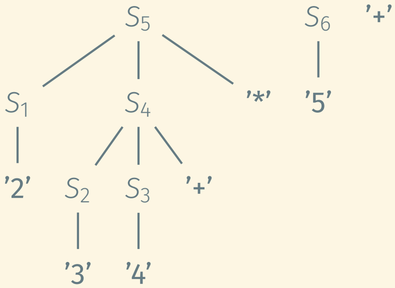
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
	$S_5\ S_6$ +

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ '+'

$S \rightarrow S\ S$ '-'

$S \rightarrow S\ S$ '*'

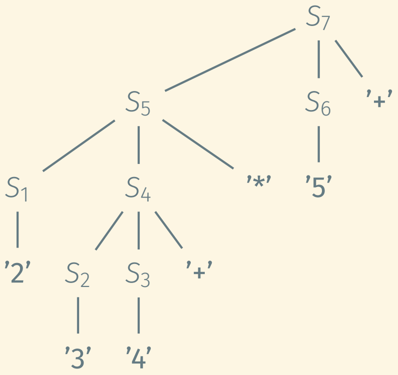
$S \rightarrow S\ S$ '/'

$S \rightarrow S$ 'neg'

$S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



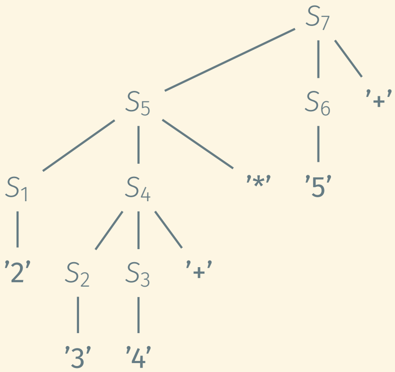
Input string	Stack
	$S_5\ S_6$ +

A grammar for arithmetic expressions in reverse Polish notation:

- $S \rightarrow S\ S$ '+'
- $S \rightarrow S\ S$ '-'
- $S \rightarrow S\ S$ '*'
- $S \rightarrow S\ S$ '/'
- $S \rightarrow S$ 'neg'
- $S \rightarrow$ int

2 * (3 + 4) + 5 $\Rightarrow$ 2 3 4 + * 5 +

Parse tree



Input string	Stack
	S_7

A grammar for arithmetic expressions in reverse Polish notation:

$S \rightarrow S\ S$ **'+'**

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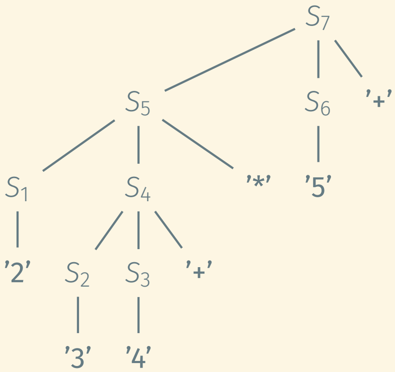

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Most efficient deterministic parsers use look-ahead to answer these questions: In each step, inspect the next k symbols in the input text and decide what to do based on these symbols.

A grammar is LL(k) if it can be parsed by a recursive-descent parser and a look-ahead of k symbols suffices to decide which production to choose when expanding a non-terminal.

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Almost every programming language can be described by an LL(1) or LR(1) grammar.

Advantages of LL parsers:

- Simpler
- Easier to understand
- More space-efficient

CHOOSING BETWEEN LL AND LR PARSERS

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Variants of LR parsers:

- SLR(1) and LALR(1)
- Less powerful than LR(1) parsers but LALR powerful enough for most programming languages
- Easier to construct than general LR parsers
- More space-efficient than general LR parsers

ROAD MAP

- **Parsing:** Transform (tokenized) program text into parse tree
- **Modelling programming languages:** Context-free grammars and languages
- **Capturing the syntactic structure of a program:** Parse trees

- Types of parsers and types of grammars they can parse
- Grammars that describe programming languages and can be parsed efficiently

- Construction of an LL(1) grammar
- Parsing LL(1) languages
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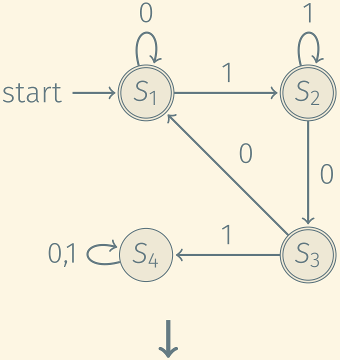
Regular grammars are too weak to express programming languages!

From DFA to right-linear grammar:

$$D = (S, \Sigma, \delta, s_0, F) \rightarrow G = (S, \Sigma, P, s_0)$$



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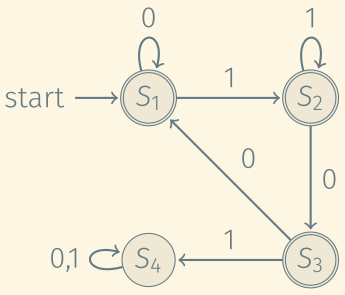

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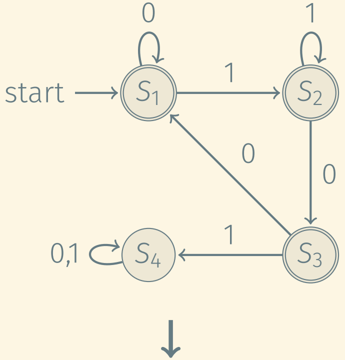

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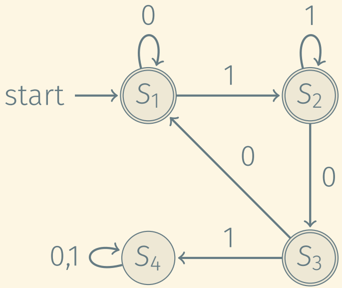

↓

$S_1 \rightarrow 0S_1$	$S_1 \rightarrow 1S_2$
$S_2 \rightarrow 0S_3$	$S_2 \rightarrow 1S_2$
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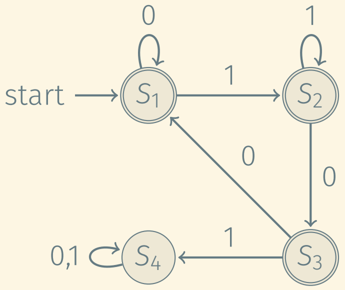

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RIGHT-LINEAR GRAMMARS MODEL REGULAR LANGUAGES (1)

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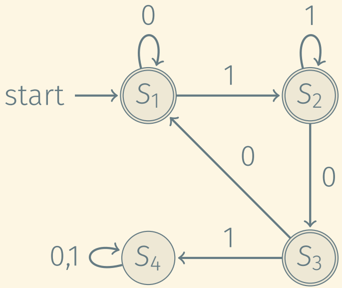

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From right-linear grammar to simplified right-linear grammar:

$$G = (V, \Sigma, P, S) \rightarrow G' = (V', \Sigma, P', S)$$

$$L \rightarrow 0L \qquad L \rightarrow 1L \qquad L \rightarrow M$$
$$M \rightarrow 101R$$
$$R \rightarrow 0R \qquad R \rightarrow 1R \qquad R \rightarrow \epsilon$$

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RIGHT-LINEAR GRAMMARS MODEL REGULAR LANGUAGES (2)

From right-linear grammar to simplified right-linear grammar:

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$$V' = V \cup \bigcup_{p \in P} V_p, \qquad P' = \bigcup_{p \in P} P_p$$

For $p = (X \to [x][Y]) \in P$,

$$V_p = \{X\}, \qquad P_p = p.$$

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For $p = (X \rightarrow x_1 x_2 \ldots x_k[Y]) \in P$, $k \geq 2$,

$$V_p = \{X = X_{p,1}, X_{p,2}, x_{p,3}, \ldots, X_{p,k}\}$$

$$P_p = \{X_{p,i} \rightarrow x_i X_{p,i+1} \mid 1 \leq i \leq k\} \cup \{X_{p,k} \rightarrow [Y]\}$$

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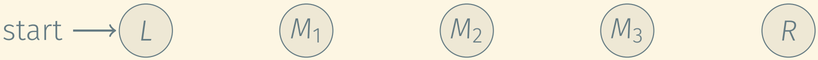

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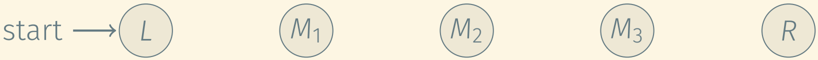

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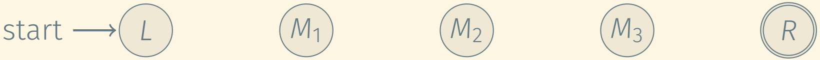

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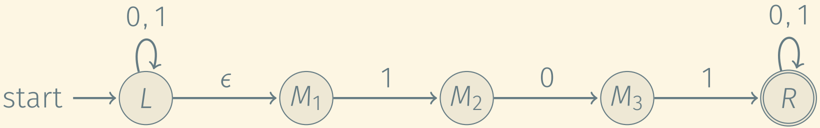

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$\mathcal{L}$ is regular if and only if $\overleftarrow{\mathcal{L}}$ is regular.

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$$L \to L0 \qquad L \to L1 \qquad L \to M$$
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S-GRAMMARS

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A non-simple context-free grammar:

$$A \rightarrow aAA$$
$$A \rightarrow a$$

EXAMPLE OF AN S-GRAMMAR WE HAVE ALREADY SEEN

An S-grammar for arithmetic expressions in Polish notation:

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Recursive-descent parser for S-grammars:

When expanding a leading non-terminal in the current sentential form, use the rule that starts with the next terminal in the input.

PARSING USING LL(1) PARSERS

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Rule	Predictor set
$S \rightarrow$ + S S	{+}
$S \rightarrow$ - S S	{-}
$S \rightarrow$ * S S	{*}
$S \rightarrow$ / S S	{/}
$S \rightarrow$ neg S	{neg}
$S \rightarrow$ int	{int}

ROAD MAP

- **Parsing:** Transform (tokenized) program text into parse tree
- **Modelling programming languages:** Context-free grammars and languages
- **Capturing the syntactic structure of a program:** Parse trees

- Types of parsers and types of grammars they can parse
- Grammars that describe programming languages and can be parsed efficiently

- Construction of an LL(1) grammar
- Parsing LL(1) languages
- Push-down automata

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COMPUTING PREDICTOR SETS

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  $a \in$ PREDICT($A \rightarrow \alpha$) if
  - $a \in$ FIRST(α) $\setminus \{\epsilon\}$ or
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We compute $\text{FIRST}(X)$ only for $X \in V \cup \Sigma$ and generate $\text{FIRST}(\sigma)$ on the fly for some strings $\sigma \in (V \cup \Sigma)^*$ as needed.

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COMPUTING FIRST

Computing FIRST(X), for $X \in V \cup \Sigma$

- For $a \in \Sigma$, FIRST(a) $= \{a\}$.
- For $X \in V$, FIRST(X) $= \emptyset$.
- Repeat until no set FIRST(X) changes for any $X \in V$:
    - FIRST(X) $=$ FIRST(X) $\cup$ FIRST($Y_1Y_2\ldots Y_k$) for each production $X \to Y_1Y_2\ldots Y_k$.

COMPUTING FIRST

Computing FIRST(X), for $X \in V \cup \boldsymbol{\Sigma}$

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Computing FIRST($Y_1 Y_2 \ldots Y_k$) on the fly

- $\text{FIRST}(Y_1 Y_2 \ldots Y_K) = \emptyset$.
- For $i = 1, 2, \ldots, k$:
  - $\text{FIRST}(Y_1 Y_2 \ldots Y_k) = \text{FIRST}(Y_1 Y_2 \ldots Y_k) \cup (\text{FIRST}(Y_i) \setminus \{\epsilon\})$
  - IF $\epsilon \notin \text{FIRST}(Y_i)$, then return.
- $\text{FIRST}(Y_1 Y_2 \ldots Y_k) = \text{FIRST}(Y_1 Y_2 \ldots Y_k) \cup \{\epsilon\}$.

COMPUTING FIRST: EXAMPLE

Is the following grammar LL(1)?

$T \rightarrow A\ B$

$A \rightarrow P\ Q$

$A \rightarrow B\ C$

$P \rightarrow p\ P$

$P \rightarrow \epsilon$

$Q \rightarrow q\ Q$

$Q \rightarrow \epsilon$

$B \rightarrow b\ B$

$B \rightarrow e$

$C \rightarrow c\ C$

$C \rightarrow f$

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X	FIRST(X)			
	Iteration 1	Iteration 2	Iteration 3	Iteration 4
p				
q				
b				
e				
c				
f				
T				
A				
P				
Q				
B				
C				

COMPUTING FIRST: EXAMPLE

Is the following grammar LL(1)?

$T \to A\ B$

$A \to P\ Q$

$A \to B\ C$

$P \to p\ P$

$P \to \epsilon$

$Q \to q\ Q$

$Q \to \epsilon$

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$B \to e$

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X	FIRST(X)			
	Iteration 1	Iteration 2	Iteration 3	Iteration 4
p	$\{p\}$			
q	$\{q\}$			
b	$\{b\}$			
e	$\{e\}$			
c	$\{c\}$			
f	$\{f\}$			
T	$\emptyset$			
A	$\emptyset$			
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T	$\emptyset$	$\emptyset$	$\emptyset$	
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Q	$\emptyset$	$\{q, \epsilon\}$	$\{q, \epsilon\}$	
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	Iteration 1	Iteration 2	Iteration 3	Iteration 4
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A	$\emptyset$	$\emptyset$	$\{p, q, \epsilon, b, e\}$	$\{p, q, \epsilon, b, e\}$
P	$\emptyset$	$\{p, \epsilon\}$	$\{p, \epsilon\}$	$\{p, \epsilon\}$
Q	$\emptyset$	$\{q, \epsilon\}$	$\{q, \epsilon\}$	$\{q, \epsilon\}$
B	$\emptyset$	$\{b, e\}$	$\{b, e\}$	$\{b, e\}$
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COMPUTING PREDICTOR SETS

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- FOLLOW(X) $\subseteq \Sigma \cup \{\epsilon\}$, for all $X \in V$:
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  $a \in$ PREDICT($A \rightarrow \alpha$) if
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  - $\epsilon \in$ FIRST(α) and $a \in$ FOLLOW(A).

COMPUTING FOLLOW

- $\text{FOLLOW}(S) = \{\epsilon\}$.
- $\text{FOLLOW}(X) = \emptyset$ for all $X \in V \setminus \{S\}$.
- Repeat until no set $\text{FOLLOW}(X)$ changes:
  - For each production $A \rightarrow \alpha B \beta$:
    - $\text{FOLLOW}(B) = \text{FOLLOW}(B) \cup (\text{FIRST}(\beta) \setminus \{\epsilon\})$.
    - If $\epsilon \in \text{FIRST}(\beta)$, then $\text{FOLLOW}(B) = \text{FOLLOW}(B) \cup \text{FOLLOW}(A)$.

Is the following grammar LL(1)?

$T \rightarrow A\ B$

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$B \rightarrow e$

$C \rightarrow c\ C$

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COMPUTING FOLLOW: EXAMPLE

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$Q \rightarrow q\ Q$

$Q \rightarrow \epsilon$

$B \rightarrow b\ B$

$B \rightarrow e$

$C \rightarrow c\ C$

$C \rightarrow f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

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X	FOLLOW(X) Iter 1	FOLLOW(X) Iter 2	FOLLOW(X) Iter 3
T			
A			
P			
Q			
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X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
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T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$		
A	$\emptyset$		
P	$\emptyset$		
Q	$\emptyset$		
B	$\emptyset$		
C	$\emptyset$		

COMPUTING FOLLOW: EXAMPLE

Is the following grammar LL(1)?

$T \rightarrow A\ B$

$A \rightarrow P\ Q$

$A \rightarrow B\ C$

$P \rightarrow p\ P$

$P \rightarrow \epsilon$

$Q \rightarrow q\ Q$

$Q \rightarrow \epsilon$

$B \rightarrow b\ B$

$B \rightarrow e$

$C \rightarrow c\ C$

$C \rightarrow f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X) Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$		
P	$\emptyset$		
Q	$\emptyset$		
B	$\emptyset$		
C	$\emptyset$		

Is the following grammar LL(1)?

$T \to A\ B$

$A \to P\ Q$

$A \to B\ C$

$P \to p\ P$

$P \to \epsilon$

$Q \to q\ Q$

$Q \to \epsilon$

$B \to b\ B$

$B \to e$

$C \to c\ C$

$C \to f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$	$\{b, e\}$	
P	$\emptyset$		
Q	$\emptyset$		
B	$\emptyset$		
C	$\emptyset$		

Is the following grammar LL(1)?

$T \rightarrow A\ B$

$A \rightarrow P\ Q$

$A \rightarrow B\ C$

$P \rightarrow p\ P$

$P \rightarrow \epsilon$

$Q \rightarrow q\ Q$

$Q \rightarrow \epsilon$

$B \rightarrow b\ B$

$B \rightarrow e$

$C \rightarrow c\ C$

$C \rightarrow f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$	$\{b, e\}$	$\{b, e\}$
P	$\emptyset$		
Q	$\emptyset$		
B	$\emptyset$		
C	$\emptyset$		

Is the following grammar LL(1)?

$T \rightarrow A\ B$

$A \rightarrow P\ Q$

$A \rightarrow B\ C$

$P \rightarrow p\ P$

$P \rightarrow \epsilon$

$Q \rightarrow q\ Q$

$Q \rightarrow \epsilon$

$B \rightarrow b\ B$

$B \rightarrow e$

$C \rightarrow c\ C$

$C \rightarrow f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$	$\{b, e\}$	$\{b, e\}$
P	$\emptyset$	$\{q\}$	
Q	$\emptyset$		
B	$\emptyset$		
C	$\emptyset$		

Is the following grammar LL(1)?

$T \rightarrow A\ B$

$A \rightarrow P\ Q$

$A \rightarrow B\ C$

$P \rightarrow p\ P$

$P \rightarrow \epsilon$

$Q \rightarrow q\ Q$

$Q \rightarrow \epsilon$

$B \rightarrow b\ B$

$B \rightarrow e$

$C \rightarrow c\ C$

$C \rightarrow f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$	$\{b, e\}$	$\{b, e\}$
P	$\emptyset$	$\{q\}$	
Q	$\emptyset$	$\emptyset$	
B	$\emptyset$		
C	$\emptyset$		

COMPUTING FOLLOW: EXAMPLE

Is the following grammar LL(1)?

$T \rightarrow A\ B$

$A \rightarrow P\ Q$

$A \rightarrow B\ C$

$P \rightarrow p\ P$

$P \rightarrow \epsilon$

$Q \rightarrow q\ Q$

$Q \rightarrow \epsilon$

$B \rightarrow b\ B$

$B \rightarrow e$

$C \rightarrow c\ C$

$C \rightarrow f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$	$\{b, e\}$	$\{b, e\}$
P	$\emptyset$	$\{q\}$	
Q	$\emptyset$	$\emptyset$	
B	$\emptyset$	$\{\epsilon, c, f\}$	
C	$\emptyset$		

COMPUTING FOLLOW: EXAMPLE

Is the following grammar LL(1)?

$T \to A\ B$

$A \to P\ Q$

$A \to B\ C$

$P \to p\ P$

$P \to \epsilon$

$Q \to q\ Q$

$Q \to \epsilon$

$B \to b\ B$

$B \to e$

$C \to c\ C$

$C \to f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$	$\{b, e\}$	$\{b, e\}$
P	$\emptyset$	$\{q\}$	
Q	$\emptyset$	$\emptyset$	
B	$\emptyset$	$\{\epsilon, c, f\}$	
C	$\emptyset$	$\emptyset$	

Is the following grammar LL(1)?

$T \to A\ B$

$A \to P\ Q$

$A \to B\ C$

$P \to p\ P$

$P \to \epsilon$

$Q \to q\ Q$

$Q \to \epsilon$

$B \to b\ B$

$B \to e$

$C \to c\ C$

$C \to f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$	$\{b, e\}$	$\{b, e\}$
P	$\emptyset$	$\{q\}$	$\{q, b, e\}$
Q	$\emptyset$	$\emptyset$	
B	$\emptyset$	$\{\epsilon, c, f\}$	
C	$\emptyset$	$\emptyset$	

COMPUTING FOLLOW: EXAMPLE

Is the following grammar LL(1)?

$T \rightarrow A\ B$

$A \rightarrow P\ Q$

$A \rightarrow B\ C$

$P \rightarrow p\ P$

$P \rightarrow \epsilon$

$Q \rightarrow q\ Q$

$Q \rightarrow \epsilon$

$B \rightarrow b\ B$

$B \rightarrow e$

$C \rightarrow c\ C$

$C \rightarrow f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$	$\{b, e\}$	$\{b, e\}$
P	$\emptyset$	$\{q\}$	$\{q, b, e\}$
Q	$\emptyset$	$\emptyset$	$\{b, e\}$
B	$\emptyset$	$\{\epsilon, c, f\}$	
C	$\emptyset$	$\emptyset$	

Is the following grammar LL(1)?

$T \rightarrow A\ B$

$A \rightarrow P\ Q$

$A \rightarrow B\ C$

$P \rightarrow p\ P$

$P \rightarrow \epsilon$

$Q \rightarrow q\ Q$

$Q \rightarrow \epsilon$

$B \rightarrow b\ B$

$B \rightarrow e$

$C \rightarrow c\ C$

$C \rightarrow f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$	$\{b, e\}$	$\{b, e\}$
P	$\emptyset$	$\{q\}$	$\{q, b, e\}$
Q	$\emptyset$	$\emptyset$	$\{b, e\}$
B	$\emptyset$	$\{\epsilon, c, f\}$	$\{\epsilon, c, f\}$
C	$\emptyset$	$\emptyset$	

COMPUTING FOLLOW: EXAMPLE

Is the following grammar LL(1)?

$T \to A\ B$

$A \to P\ Q$

$A \to B\ C$

$P \to p\ P$

$P \to \epsilon$

$Q \to q\ Q$

$Q \to \epsilon$

$B \to b\ B$

$B \to e$

$C \to c\ C$

$C \to f$

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)		
	Iter 1	Iter 2	Iter 3
T	$\{\epsilon\}$	$\{\epsilon\}$	$\{\epsilon\}$
A	$\emptyset$	$\{b, e\}$	$\{b, e\}$
P	$\emptyset$	$\{q\}$	$\{q, b, e\}$
Q	$\emptyset$	$\emptyset$	$\{b, e\}$
B	$\emptyset$	$\{\epsilon, c, f\}$	$\{\epsilon, c, f\}$
C	$\emptyset$	$\emptyset$	$\{b, e\}$

Compute three kinds of sets:

- $\text{FIRST}(\sigma) \subseteq \Sigma \cup \{\epsilon\}$, for all $\sigma \in (V \cup \Sigma)^*$:
  - For $a \in \Sigma$, $a \in \text{FIRST}(\sigma)$ if $\sigma \Rightarrow^* a\beta$.
  - $\epsilon \in \text{FIRST}(\sigma)$ if $\sigma \Rightarrow^* \epsilon$.

- $\text{FOLLOW}(X) \subseteq \Sigma \cup \{\epsilon\}$, for all $X \in V$:
  - For $a \in \Sigma$, $a \in \text{FOLLOW}(X)$ if $S \Rightarrow^* \alpha X a \beta$.
  - $\epsilon \in \text{FOLLOW}(X)$ if $S \Rightarrow^* \alpha X$.

- $\text{PREDICT}(A \rightarrow \alpha) \subseteq \Sigma \cup \{\epsilon\}$, for all $(A \rightarrow \alpha) \in P$:

  $a \in \text{PREDICT}(A \rightarrow \alpha)$ if

  - $a \in \text{FIRST}(\alpha) \setminus \{\epsilon\}$ or
  - $\epsilon \in \text{FIRST}(\alpha)$ and $a \in \text{FOLLOW}(A)$.

COMPUTING PREDICTOR SETS

Compute three kinds of sets:

- $\text{FIRST}(\sigma) \subseteq \Sigma \cup \{\epsilon\}$, for all $\sigma \in (V \cup \Sigma)^*$:
  - For $a \in \Sigma$, $a \in \text{FIRST}(\sigma)$ if $\sigma \Rightarrow^* a\beta$.
  - $\epsilon \in \text{FIRST}(\sigma)$ if $\sigma \Rightarrow^* \epsilon$.

- $\text{FOLLOW}(X) \subseteq \Sigma \cup \{\epsilon\}$, for all $X \in V$:
  - For $a \in \Sigma$, $a \in \text{FOLLOW}(X)$ if $S \Rightarrow^* \alpha X a \beta$.
  - $\epsilon \in \text{FOLLOW}(X)$ if $S \Rightarrow^* \alpha X$.

- $\text{PREDICT}(A \rightarrow \alpha) \subseteq \Sigma \cup \{\epsilon\}$, for all $(A \rightarrow \alpha) \in P$:

  $a \in \text{PREDICT}(A \rightarrow \alpha)$ if

  - $a \in \text{FIRST}(\alpha) \setminus \{\epsilon\}$ or
  - $\epsilon \in \text{FIRST}(\alpha)$ and $a \in \text{FOLLOW}(A)$.

- For every production $A \rightarrow \alpha$:
  - $\text{PREDICT}(A \rightarrow \alpha) = \text{FIRST}(\alpha) \setminus \{\epsilon\}$.
  - If $\epsilon \in \text{FIRST}(\alpha)$, then $\text{PREDICT}(A \rightarrow \alpha) = \text{PREDICT}(A \rightarrow \alpha) \cup \text{FOLLOW}(A)$.

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

Rule R	PREDICT(R)
$T \rightarrow A\ B$	
$A \rightarrow P\ Q$	
$A \rightarrow B\ C$	
$P \rightarrow p\ P$	
$P \rightarrow \epsilon$	
$Q \rightarrow q\ Q$	
$Q \rightarrow \epsilon$	
$B \rightarrow b\ B$	
$B \rightarrow e$	
$C \rightarrow c\ C$	
$C \rightarrow f$	

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

Rule R	PREDICT(R)
$T \rightarrow A\ B$	$\{p, q, b, e\}$
$A \rightarrow P\ Q$	
$A \rightarrow B\ C$	
$P \rightarrow p\ P$	
$P \rightarrow \epsilon$	
$Q \rightarrow q\ Q$	
$Q \rightarrow \epsilon$	
$B \rightarrow b\ B$	
$B \rightarrow e$	
$C \rightarrow c\ C$	
$C \rightarrow f$	

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

Rule R	PREDICT(R)
$T \rightarrow A\ B$	$\{p, q, b, e\}$
$A \rightarrow P\ Q$	$\{p, q, b, e\}$
$A \rightarrow B\ C$	
$P \rightarrow p\ P$	
$P \rightarrow \epsilon$	
$Q \rightarrow q\ Q$	
$Q \rightarrow \epsilon$	
$B \rightarrow b\ B$	
$B \rightarrow e$	
$C \rightarrow c\ C$	
$C \rightarrow f$	

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

Rule R	PREDICT(R)
$T \to A\ B$	$\{p, q, b, e\}$
$A \to P\ Q$	$\{p, q, b, e\}$
$A \to B\ C$	$\{b, e\}$
$P \to p\ P$	
$P \to \epsilon$	
$Q \to q\ Q$	
$Q \to \epsilon$	
$B \to b\ B$	
$B \to e$	
$C \to c\ C$	
$C \to f$	

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

Rule R	PREDICT(R)
$T \rightarrow A\ B$	$\{p, q, b, e\}$
$A \rightarrow P\ Q$	$\{p, q, b, e\}$
$A \rightarrow B\ C$	$\{b, e\}$
$P \rightarrow p\ P$	$\{p\}$
$P \rightarrow \epsilon$	
$Q \rightarrow q\ Q$	
$Q \rightarrow \epsilon$	
$B \rightarrow b\ B$	
$B \rightarrow e$	
$C \rightarrow c\ C$	
$C \rightarrow f$	

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

Rule R	PREDICT(R)
$T \rightarrow A\ B$	$\{p, q, b, e\}$
$A \rightarrow P\ Q$	$\{p, q, b, e\}$
$A \rightarrow B\ C$	$\{b, e\}$
$P \rightarrow p\ P$	$\{p\}$
$P \rightarrow \epsilon$	$\{q, b, e\}$
$Q \rightarrow q\ Q$	
$Q \rightarrow \epsilon$	
$B \rightarrow b\ B$	
$B \rightarrow e$	
$C \rightarrow c\ C$	
$C \rightarrow f$	

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

Rule R	PREDICT(R)
$T \rightarrow A\ B$	$\{p, q, b, e\}$
$A \rightarrow P\ Q$	$\{p, q, b, e\}$
$A \rightarrow B\ C$	$\{b, e\}$
$P \rightarrow p\ P$	$\{p\}$
$P \rightarrow \epsilon$	$\{q, b, e\}$
$Q \rightarrow q\ Q$	$\{q\}$
$Q \rightarrow \epsilon$	
$B \rightarrow b\ B$	
$B \rightarrow e$	
$C \rightarrow c\ C$	
$C \rightarrow f$	

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

Rule R	PREDICT(R)
$T \to A\ B$	$\{p, q, b, e\}$
$A \to P\ Q$	$\{p, q, b, e\}$
$A \to B\ C$	$\{b, e\}$
$P \to p\ P$	$\{p\}$
$P \to \epsilon$	$\{q, b, e\}$
$Q \to q\ Q$	$\{q\}$
$Q \to \epsilon$	$\{b, e\}$
$B \to b\ B$	
$B \to e$	
$C \to c\ C$	
$C \to f$	

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

Rule R	PREDICT(R)
$T \rightarrow A\ B$	$\{p, q, b, e\}$
$A \rightarrow P\ Q$	$\{p, q, b, e\}$
$A \rightarrow B\ C$	$\{b, e\}$
$P \rightarrow p\ P$	$\{p\}$
$P \rightarrow \epsilon$	$\{q, b, e\}$
$Q \rightarrow q\ Q$	$\{q\}$
$Q \rightarrow \epsilon$	$\{b, e\}$
$B \rightarrow b\ B$	$\{b\}$
$B \rightarrow e$	
$C \rightarrow c\ C$	
$C \rightarrow f$	

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

Rule R	PREDICT(R)
$T \to A\ B$	$\{p, q, b, e\}$
$A \to P\ Q$	$\{p, q, b, e\}$
$A \to B\ C$	$\{b, e\}$
$P \to p\ P$	$\{p\}$
$P \to \epsilon$	$\{q, b, e\}$
$Q \to q\ Q$	$\{q\}$
$Q \to \epsilon$	$\{b, e\}$
$B \to b\ B$	$\{b\}$
$B \to e$	$\{e\}$
$C \to c\ C$	
$C \to f$	

COMPUTING PREDICT: EXAMPLE

X	FIRST(X)
p	$\{p\}$
q	$\{q\}$
b	$\{b\}$
e	$\{e\}$
c	$\{c\}$
f	$\{f\}$
T	$\{p, q, b, e\}$
A	$\{p, q, \epsilon, b, e\}$
P	$\{p, \epsilon\}$
Q	$\{q, \epsilon\}$
B	$\{b, e\}$
C	$\{c, f\}$

X	FOLLOW(X)
T	$\{\epsilon\}$
A	$\{b, e\}$
P	$\{q, b, e\}$
Q	$\{b, e\}$
B	$\{\epsilon, c, f\}$
C	$\{b, e\}$

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A	$\{b, e\}$
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B	$\{\epsilon, c, f\}$
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$P \rightarrow p\ P$	$\{p\}$
$P \rightarrow \epsilon$	$\{q, b, e\}$
$Q \rightarrow q\ Q$	$\{q\}$
$Q \rightarrow \epsilon$	$\{b, e\}$
$B \rightarrow b\ B$	$\{b\}$
$B \rightarrow e$	$\{e\}$
$C \rightarrow c\ C$	$\{c\}$
$C \rightarrow f$	$\{f\}$

This grammar is not LL(1)!

There exist context-free languages that do not have LL(1) grammars.

SOME FACTS ABOUT LL(1) LANGUAGES

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There is no known algorithm to determine whether a language is LL(1) (but there is an algorithm to decide whether a grammar is LL(1)).

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The "obvious" grammar for most programming languages is usually not LL(1).

In many situations, a non-LL(1) grammar can be transformed into an LL(1) grammar for the same language.

CONVERTING A GRAMMAR TO LL(1)

Two common reasons why a grammar is not LL(1) are "left-recursion" and "common prefixes", both of which can be eliminated by modifying the grammar.

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Left recursion:

$$A \to \alpha | A\beta$$

Common prefix:

$$A \to \alpha\beta | \alpha\gamma$$

Example of a common prefix:

$$Expr \to Term$$

$$Expr \to Term + Expr$$

REMOVING LEFT-RECURSION

Left-recursion can be replaced with right-recursion:

$$A \rightarrow \alpha | A\beta$$

$$\Downarrow$$

$$A \rightarrow \alpha A'$$

$$A' \rightarrow \epsilon | \beta A'$$

Caveat:

- Left-recursion is often used intentionally to capture the structure of the language (e.g., associativity of operators in arithmetic expressions).
- The above conversion discards this information.

REMOVING COMMON PREFIXES

Common prefixes can be removed using left-factoring:

$$A \rightarrow \alpha\beta|\alpha\gamma$$

$$\Downarrow$$

$$A \rightarrow \alpha A'$$

$$A' \rightarrow \beta|\gamma$$

CONVERTING A GRAMMAR TO LL(1): EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\$$	$\{n, (\}$
$E \rightarrow E\ A\ T$	$\{n, (\}$
$E \rightarrow T$	$\{n, (\}$
$T \rightarrow T\ M\ F$	$\{n, (\}$
$T \rightarrow F$	$\{n, (\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (\ E\ )$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
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CONVERTING A GRAMMAR TO LL(1): EXAMPLE

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$T \to T\ M\ F$	$\{n, (\}$
$T \to F$	$\{n, (\}$
$F \to n$	$\{n\}$
$F \to (\ E\ )$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
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Rule R	PREDICT(R)
$S \to E\$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
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ROAD MAP

- **Parsing:** Transform (tokenized) program text into parse tree
- **Modelling programming languages:** Context-free grammars and languages
- **Capturing the syntactic structure of a program:** Parse trees

- Types of parsers and types of grammars they can parse
- Grammars that describe programming languages and can be parsed efficiently

- Construction of an LL(1) grammar
- Parsing LL(1) languages
- Push-down automata

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LL(1) languages can be parsed using efficient, easy-to-implement parsers.

PARSING LL(1) LANGUAGES

LL(1) languages can be parsed using efficient, easy-to-implement parsers.

Two approaches:

- Recursive-descent parser
- Deterministic push-down automaton

LL(1) languages can be parsed using efficient, easy-to-implement parsers.

Two approaches:

- Recursive-descent parser
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Recursive-descent parser:

For each non-terminal X, write a produce parse$X$:

- Choose production $X \rightarrow Y_1 Y_2 \ldots Y_k$ whose predictor set contains next token.
- For $i = 1, 2, \ldots, k$:
  - If Y_i is a terminal, match Y_1 with next input token.
  - If Y_i is a non-terminal, call parseY_i.

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
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`2 * 40 - 18 * 3 $`

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$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
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2 * 40 - 18 * 3 $
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def parseS():
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def parseS():
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def parseE():
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    parseT()
    parseE'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
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$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
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$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
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$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
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    Match $

def parseE():
  if next token is n or (:
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    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
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$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

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    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
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RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
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$E \rightarrow T\ E'$	$\{n, (\}$
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$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
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```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
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$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
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```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

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def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
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def parseS():
  if next token is n or (:
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    Match $


def parseE():
  if next token is n or (:
    parseT()
    parseE'()


def parseT():
  if next token is n or (:
    parseF()
    parseT'()


def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
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    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $


def parseE():
  if next token is n or (:
    parseT()
    parseE'()


def parseT():
  if next token is n or (:
    parseF()
    parseT'()


def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

```
def parseM():
  if next token is *:
    Match *
  elif next token is /:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

```
def parseM():
  if next token is *:
    Match *
  elif next token is /:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

```
def parseM():
  if next token is *:
    Match *
  elif next token is /:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

```
def parseM():
  if next token is *:
    Match *
  elif next token is /:
    ...
```

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

```
def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()

def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

```
def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

```
def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

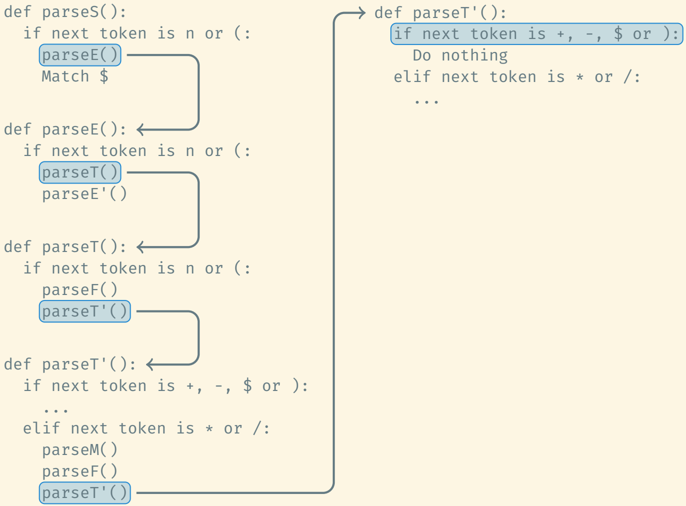
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    Do nothing
  elif next token is * or /:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

2 * 40 - 18 * 3 $



RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    Do nothing
  elif next token is * or /:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $
```

```
def parseE():
  if next token is n or (:
    parseT()
    parseE'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $


def parseE():
  if next token is n or (:
    parseT()
    parseE'()


def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseA():
  if next token is +:
    ...
  elif next token is -:
    Match -
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseA():
  if next token is +:
    ...
  elif next token is -:
    Match -
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseA():
  if next token is +:
    ...
  elif next token is -:
    Match -
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\,\$$	$\{n, (\}$
$E \rightarrow T\,E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\,T\,E'$	$\{+, -\}$
$T \rightarrow F\,T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\,F\,T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseA():
  if next token is +:
    ...
  elif next token is -:
    Match -
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseA():
  if next token is +:
    ...
  elif next token is -:
    Match -
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $


def parseE():
  if next token is n or (:
    parseT()
    parseE'()


def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $


def parseE():
  if next token is n or (:
    parseT()
    parseE'()


def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()


def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\,\$$	$\{n, (\}$
$E \rightarrow T\,E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\,T\,E'$	$\{+, -\}$
$T \rightarrow F\,T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\,F\,T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

2 * 40 - 18 * 3 $

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 [*] 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()

def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()

def parseM():
  if next token is *:
    Match *
  elif next token is /:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()

def parseM():
  if next token is *:
    Match *
  elif next token is /:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()

def parseM():
  if next token is *:
    Match *
  elif next token is /:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()

def parseM():
  if next token is *:
    Match *
  elif next token is /:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()

def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()

def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()

def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()

def parseF():
  if next token is n:
    Match n
  elif next token is (:
    ...
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $
```

```
def parseE():
  if next token is n or (:
    parseT()
    parseE'()
```

```
def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()
```

```
def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

```
2 * 40 - 18 * 3 $
```

```
def parseS():
  if next token is n or (:
    parseE()
    Match $

def parseE():
  if next token is n or (:
    parseT()
    parseE'()

def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()

def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

`2 * 40 - 18 * 3 $`

```
def parseS():
  if next token is n or (:
    parseE()
    Match $
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def parseE():
  if next token is n or (:
    parseT()
    parseE'()
```

```
def parseE'():
  if next token is $ or ):
    ...
  elif next token is + or -:
    parseA()
    parseT()
    parseE'()
```

```
def parseT():
  if next token is n or (:
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    ...
  elif next token is * or /:
    parseM()
    parseF()
    parseT'()
```

```
def parseT'():
  if next token is +, -, $ or ):
    Do nothing
  elif next token is * or /:
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RECURSIVE DESCENT PARSING: EXAMPLE

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
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$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
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    parseE'()

def parseT():
  if next token is n or (:
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    parseT'()
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def parseT'():
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    parseF()
    parseT'()

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$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
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$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
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$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
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ROAD MAP

- **Parsing:** Transform (tokenized) program text into parse tree
- **Modelling programming languages:** Context-free grammars and languages
- **Capturing the syntactic structure of a program:** Parse trees

- Types of parsers and types of grammars they can parse
- Grammars that describe programming languages and can be parsed efficiently

- Construction of an LL(1) grammar
- Parsing LL(1) languages
- Push-down automata

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We proved that a language can be parsed by a finite automaton if and only if it is regular.

PUSH-DOWN AUTOMATON: MOTIVATION

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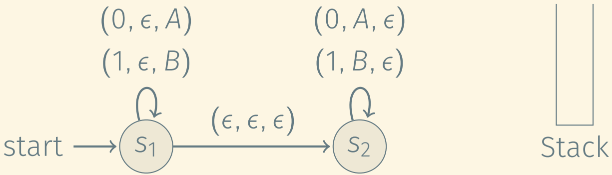
A push-down automaton (PDA) is an NFA with a stack.

Any context-free language can be parsed by a PDA.

PUSH-DOWN AUTOMATON: EXAMPLE

A PDA for the language $\{\sigma \overleftarrow{\sigma} \mid \sigma \in \{0,1\}^*\}$:



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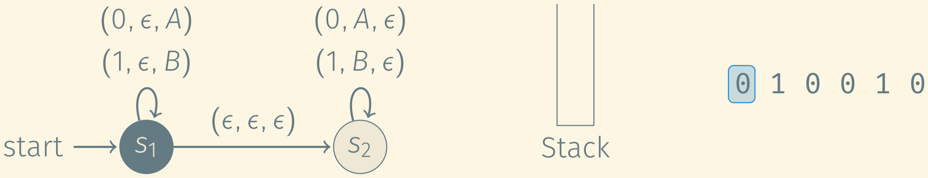

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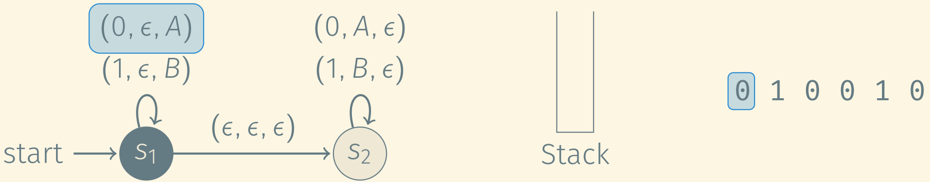

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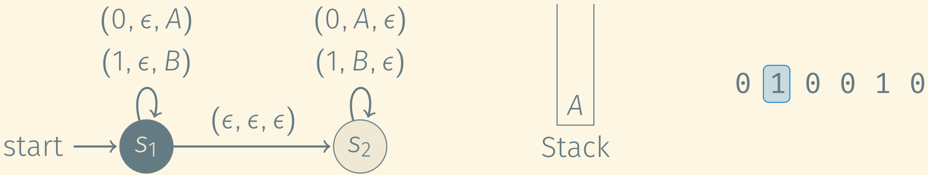

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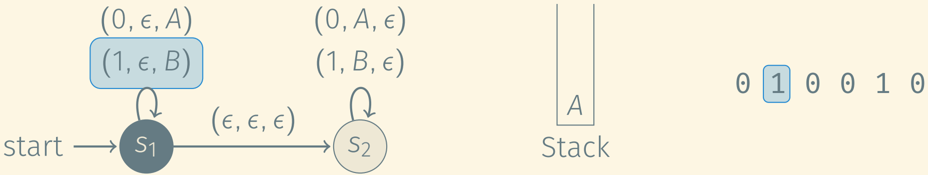

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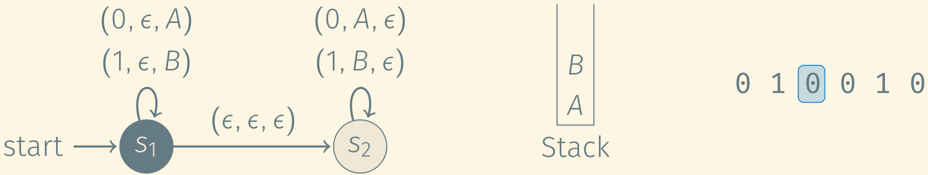

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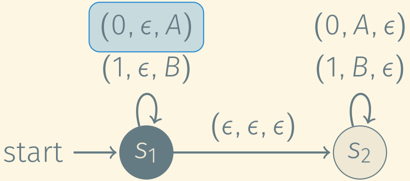

0 1 0 0 1 0

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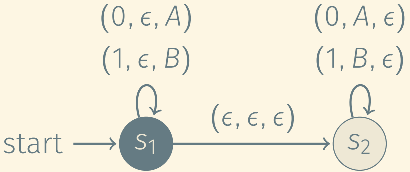

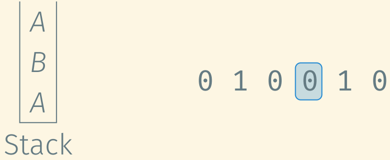

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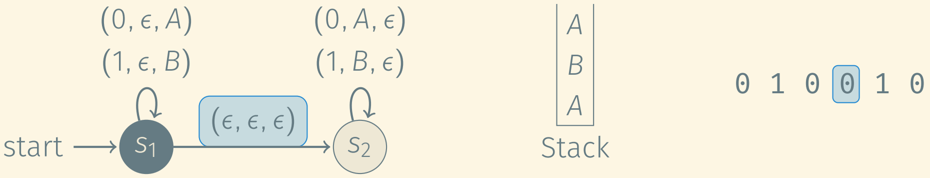

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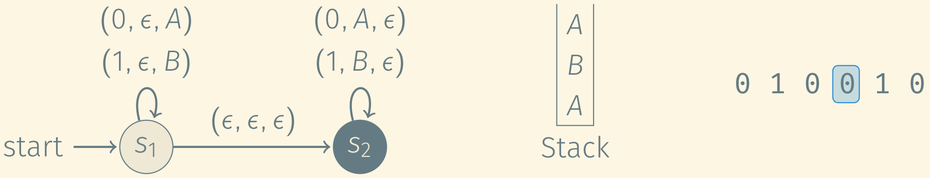

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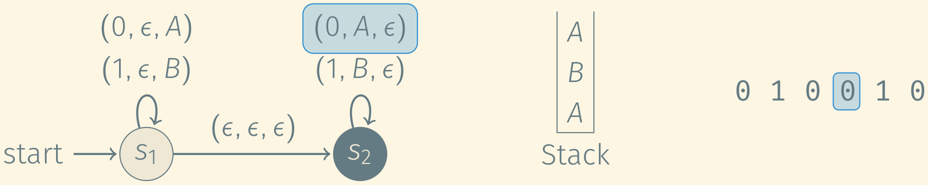

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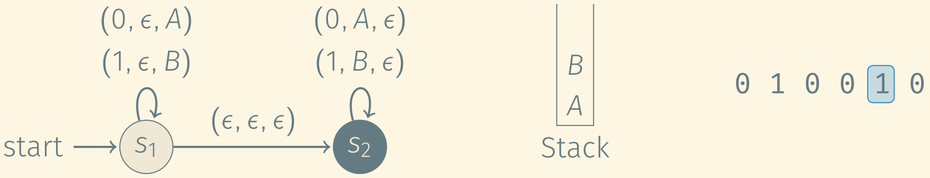

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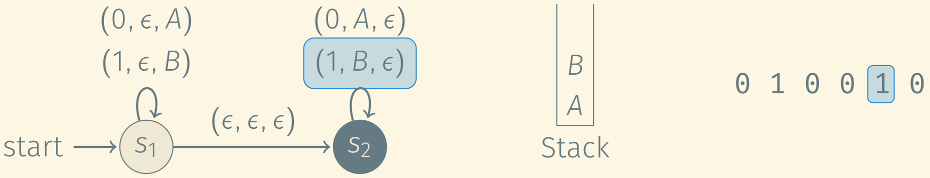

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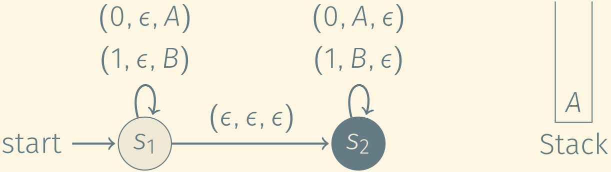

`0 1 0 0 1 0`

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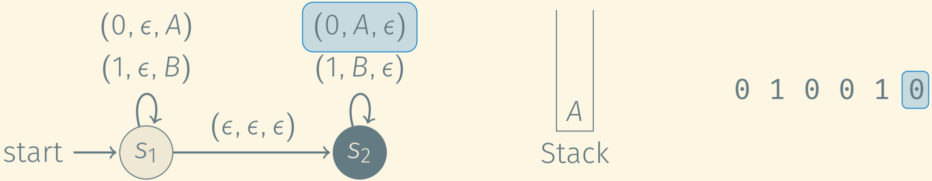

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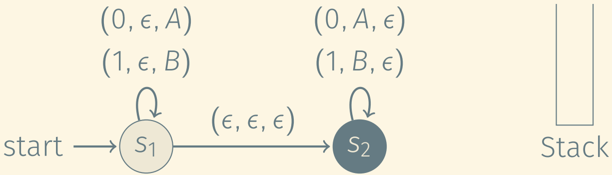

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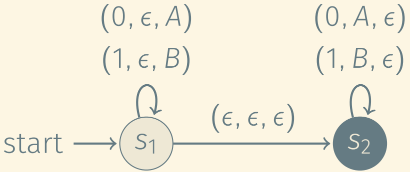

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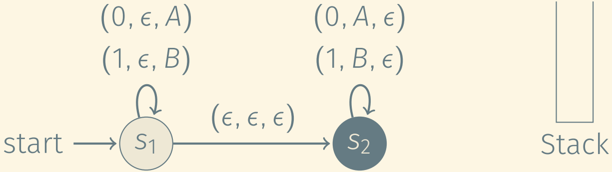

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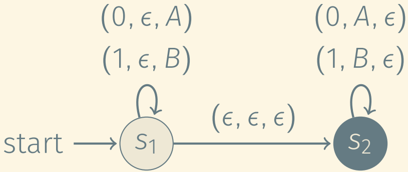

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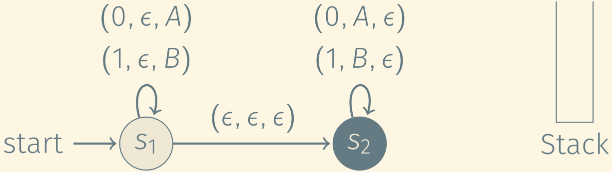

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This language cannot be parsed deterministically in a single pass!

In particular, it is not LL(k) or LR(k) for any k!

Definition: Push-down automaton (PDA)

A tuple $(S, \Sigma, \Gamma, \delta, s_0, \gamma, F)$:

- S is a finite set of states.
- Σ is the input alphabet.
- Γ is the stack alphabet.
- $\delta : S \times (\Sigma \cup \{\epsilon\}) \times (\Gamma \cup \{\epsilon\}) \to 2^{S \times \Gamma^*}$ is the transition function.
- s_0 is the start state.
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- $F \subseteq S$ is the set of accepting states.

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SOME FACTS ABOUT PDA

Lemma

The two modes of acceptance are equivalent: The exists a PDA deciding a language $\mathcal{L}$ by empty stack if and only if there exists a PDA deciding $\mathcal{L}$ by final state.

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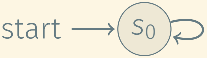

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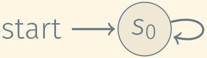

Start symbol: S

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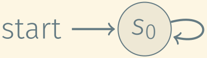

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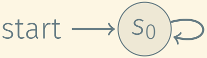

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Rules:

$$\delta(s_0, \epsilon, S) = \{(s_0, \epsilon), (s_0, (S)S)\}$$

Start symbol: S

DETERMINISTIC PUSH-DOWN AUTOMATA

By default, a PDA is non-deterministic: Multiple transitions are possible for a given combination of state, input symbol, and symbol on the top of the stack.

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A language can be decided by a DPDA if and only if it is LL(k) or LR(k) for some k.

In particular, there exist context-free languages that cannot be decided by a DPDA:

$$\{\sigma\overleftarrow{\sigma} \mid \sigma \in \{0,1\}^*\}$$

PARSING LL(1) LANGUAGES USING DPDA (1)

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
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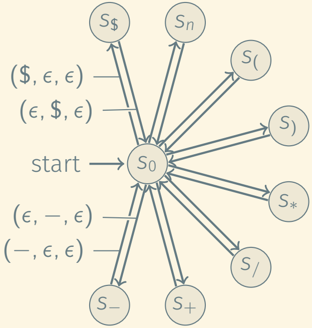

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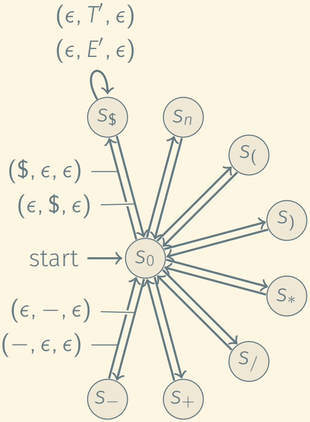

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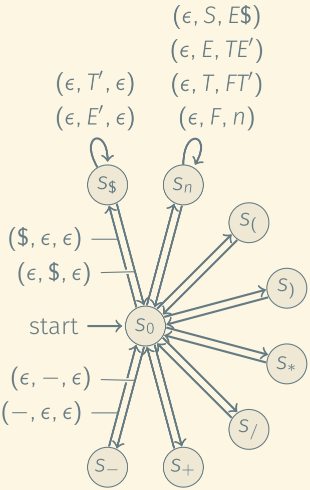

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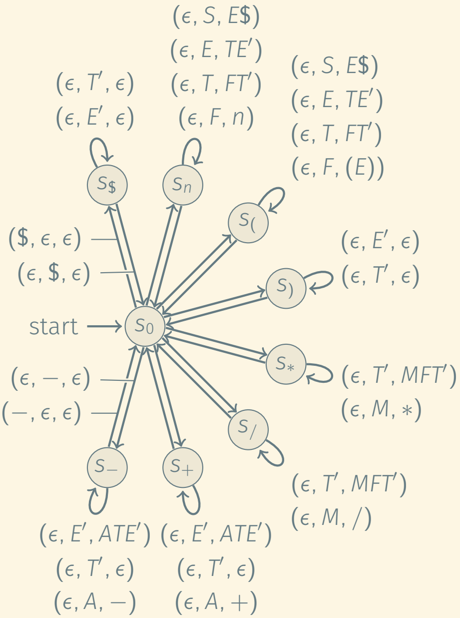

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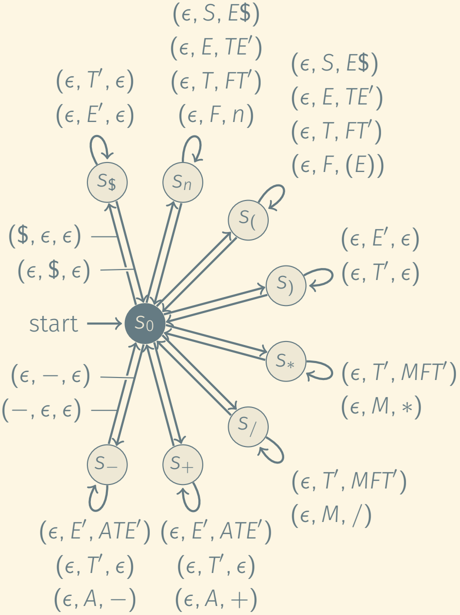

2 * 40 - 18 * 3 $

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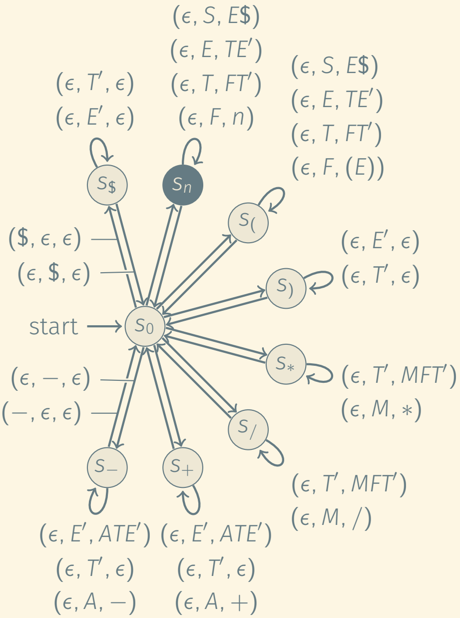

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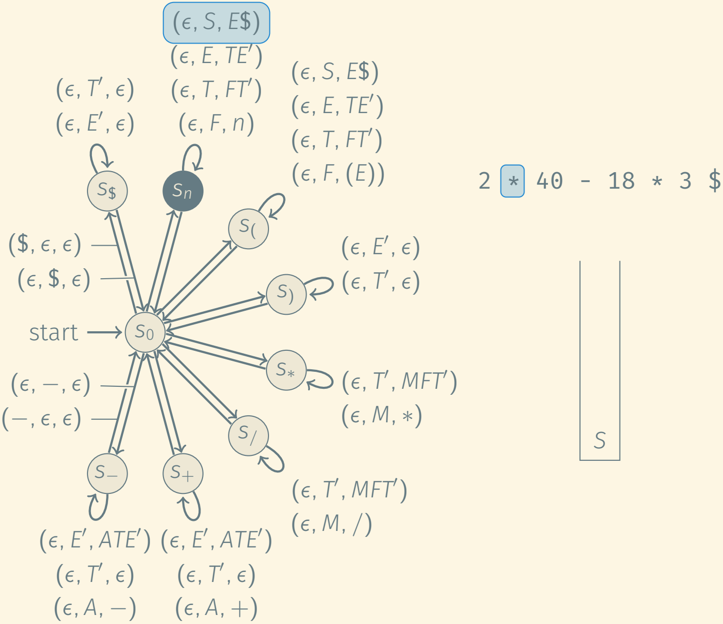

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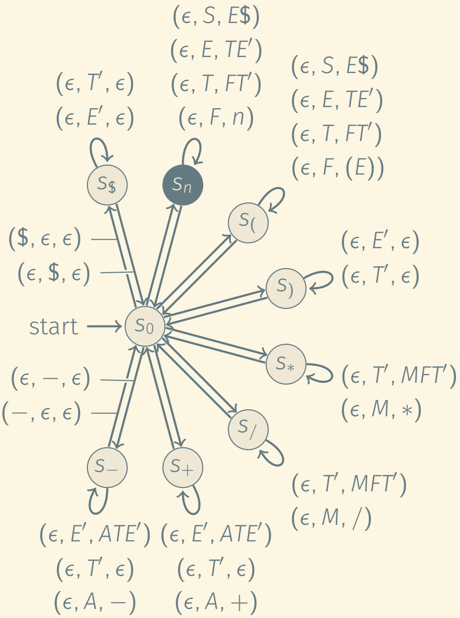

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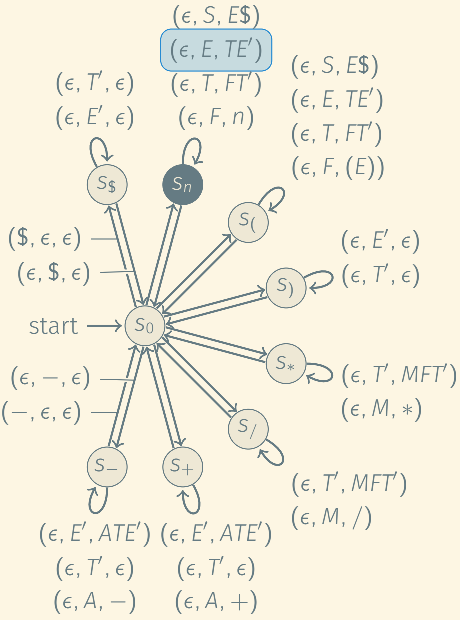

`2 * 40 - 18 * 3 $`

Stack (top to bottom):

E
$\$$

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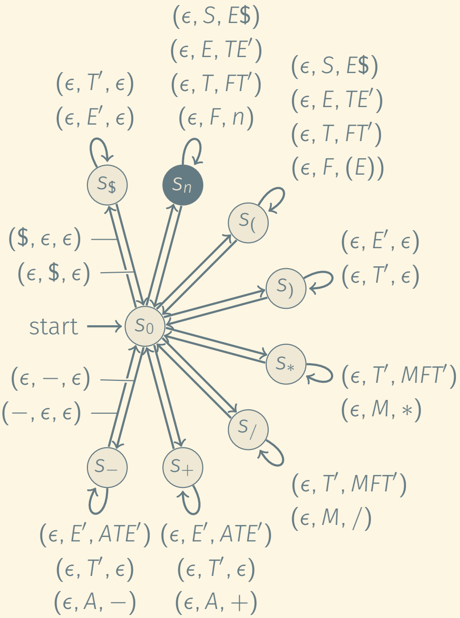

`2 * 40 - 18 * 3 $`

T
E'
$\$$

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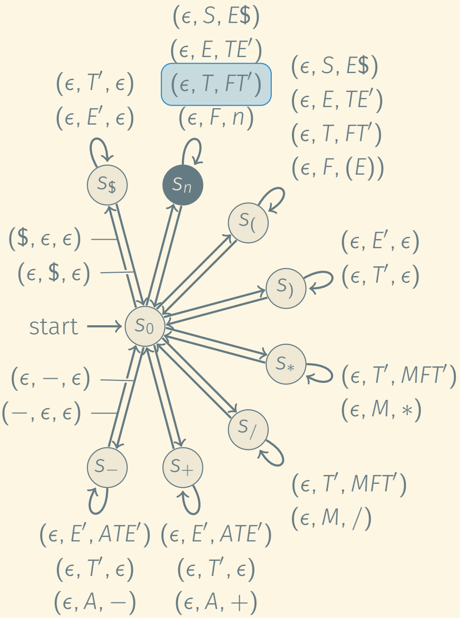

`2 * 40 - 18 * 3 $`

T
E'
$

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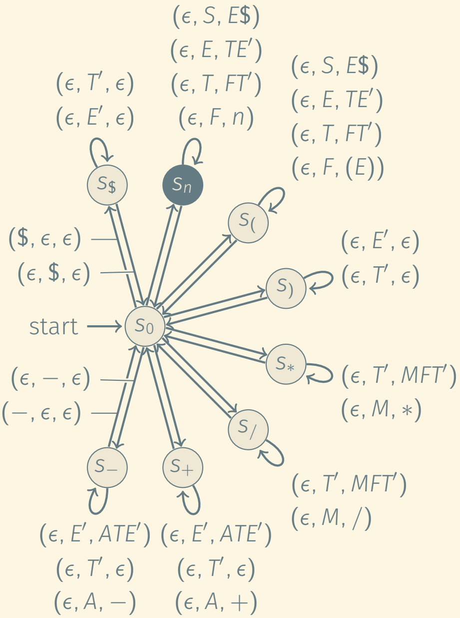
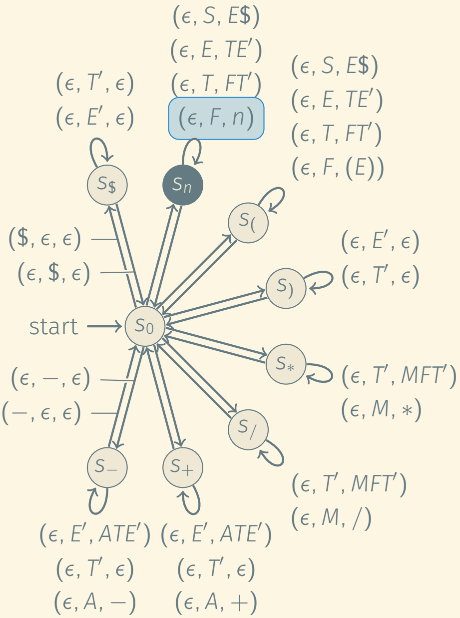
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$



2 * 40 - 18 * 3 $

F
T'
E'
$\$$

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$



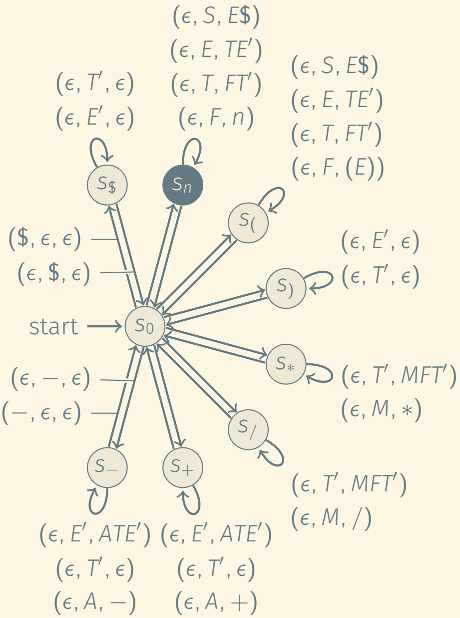
`2 * 40 - 18 * 3 $`

Stack:

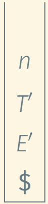
F
T'
E'
$\$$

PARSING LL(1) LANGUAGES USING DPDA (1)

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

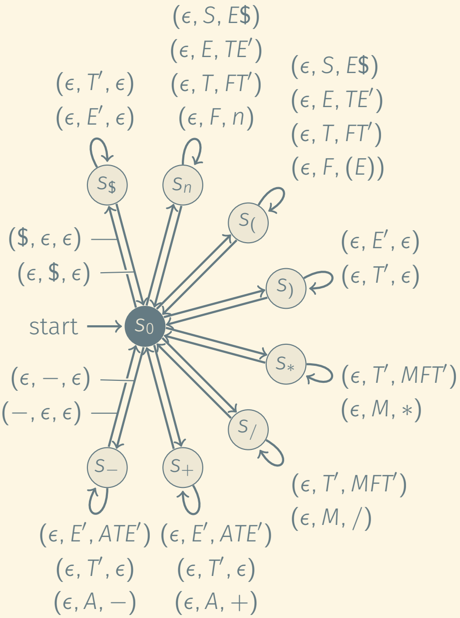



```
2 * 40 - 18 * 3 $
```

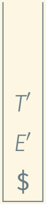


PARSING LL(1) LANGUAGES USING DPDA (1)

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$ $E' \to A\,T\,E'$	$\{\$,)\}$ $\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$ $T' \to M\,F\,T'$	$\{+, -, \$,)\}$ $\{*, /\}$
$F \to n$ $F \to (E)$	$\{n\}$ $\{(\}$
$A \to +$ $A \to -$	$\{+\}$ $\{-\}$
$M \to *$ $M \to /$	$\{*\}$ $\{/\}$

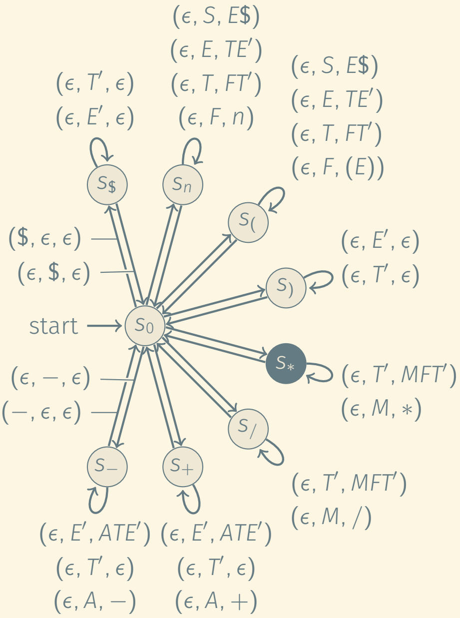


`2 * 40 - 18 * 3 $`

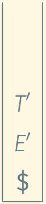


PARSING LL(1) LANGUAGES USING DPDA (1)

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$ $E' \rightarrow A\ T\ E'$	$\{\$,)\}$ $\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$ $T' \rightarrow M\ F\ T'$	$\{+, -, \$,)\}$ $\{*, /\}$
$F \rightarrow n$ $F \rightarrow (E)$	$\{n\}$ $\{(\}$
$A \rightarrow +$ $A \rightarrow -$	$\{+\}$ $\{-\}$
$M \rightarrow *$ $M \rightarrow /$	$\{*\}$ $\{/\}$

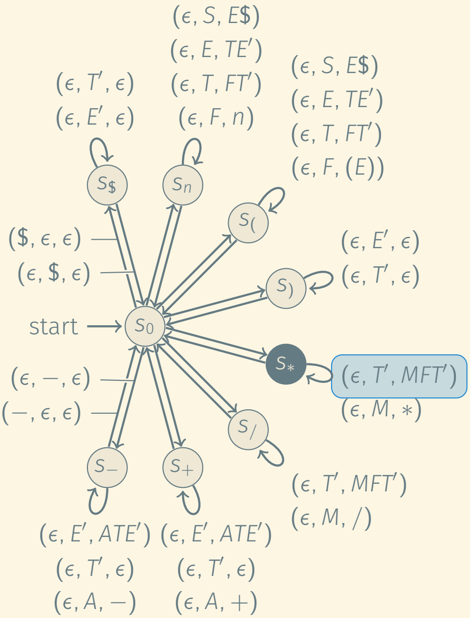


`2 * 40 - 18 * 3 $`



PARSING LL(1) LANGUAGES USING DPDA (1)

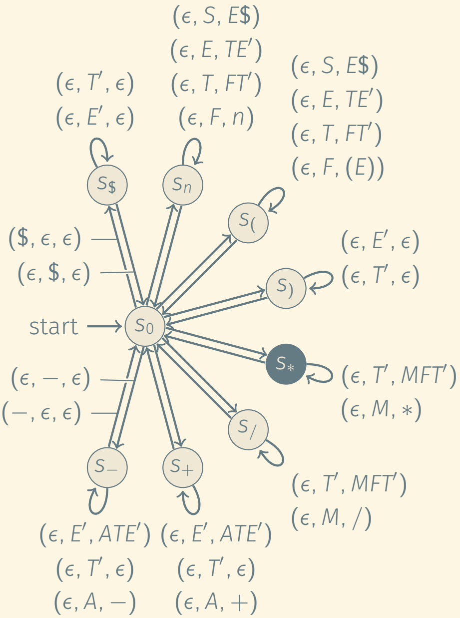
Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$ $E' \to A\ T\ E'$	$\{\$,)\}$ $\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$ $T' \to M\ F\ T'$	$\{+, -, \$,)\}$ $\{*, /\}$
$F \to n$ $F \to (E)$	$\{n\}$ $\{(\}$
$A \to +$ $A \to -$	$\{+\}$ $\{-\}$
$M \to *$ $M \to /$	$\{*\}$ $\{/\}$



`2 * 40 - 18 * 3 $`

Stack
T'
E'
$\$$

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

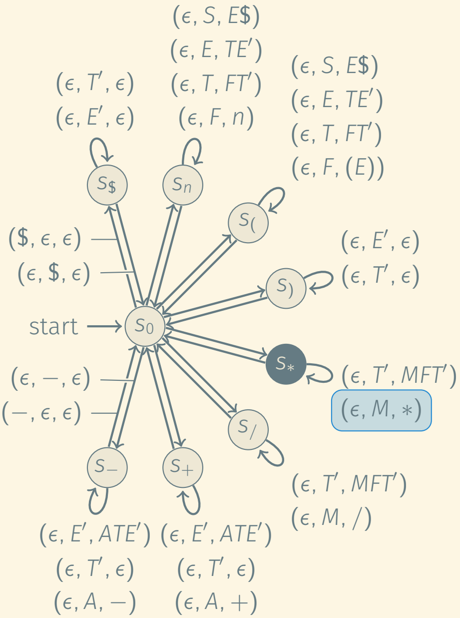


`2 * 40 - 18 * 3 $`

M
F
T'
E'
$\$$

PARSING LL(1) LANGUAGES USING DPDA (1)

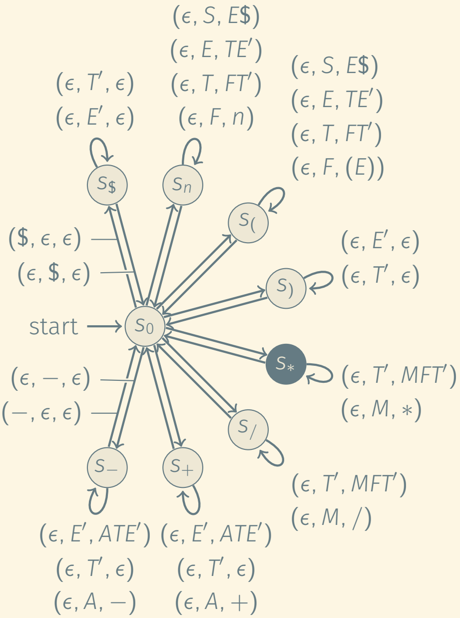
Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

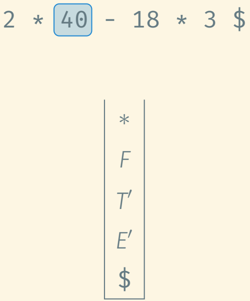


2 * 40 - 18 * 3 $

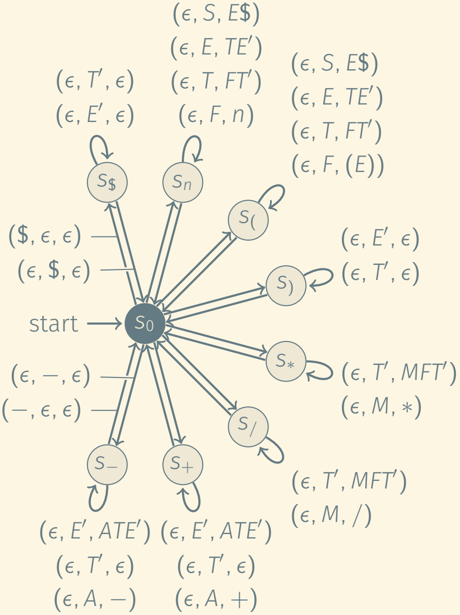
M
F
T'
E'
$\$$

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

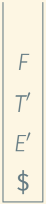




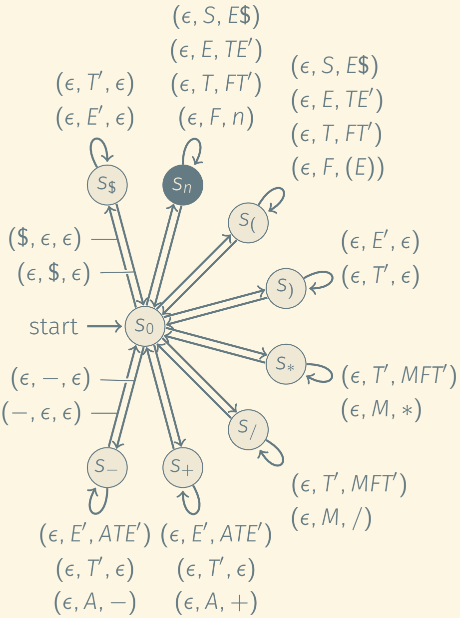
Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$



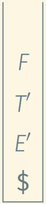
`2 * 40 - 18 * 3 $`



Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

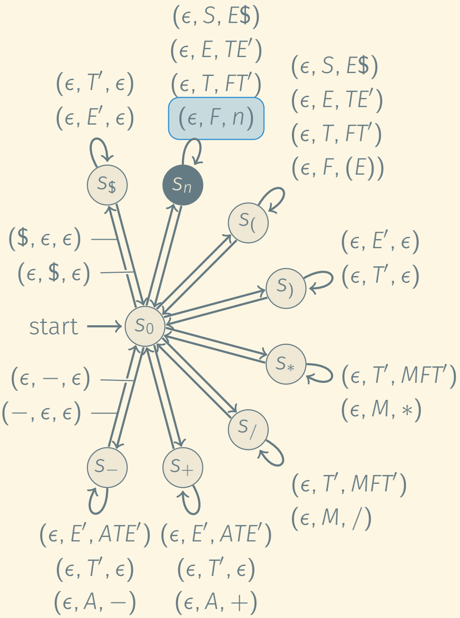


`2 * 40 - 18 * 3 $`

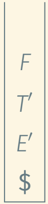


PARSING LL(1) LANGUAGES USING DPDA (1)

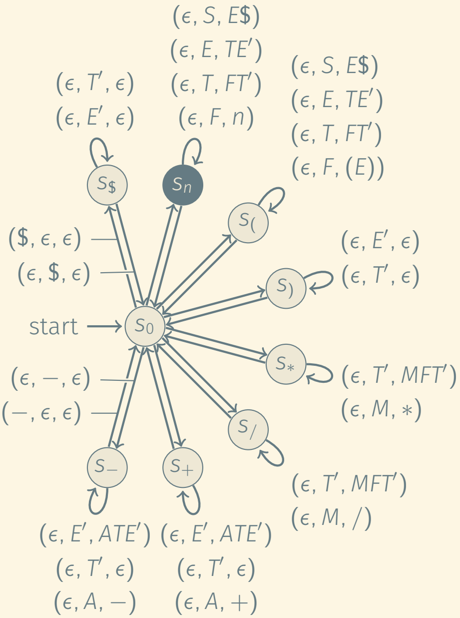
Rule R	PREDICT(R)
$S \rightarrow E\,\$$	$\{n, (\}$
$E \rightarrow T\,E'$	$\{n, (\}$
$E' \rightarrow \epsilon$ $E' \rightarrow A\,T\,E'$	$\{\$,)\}$ $\{+, -\}$
$T \rightarrow F\,T'$	$\{n, (\}$
$T' \rightarrow \epsilon$ $T' \rightarrow M\,F\,T'$	$\{+, -, \$,)\}$ $\{*, /\}$
$F \rightarrow n$ $F \rightarrow (E)$	$\{n\}$ $\{(\}$
$A \rightarrow +$ $A \rightarrow -$	$\{+\}$ $\{-\}$
$M \rightarrow *$ $M \rightarrow /$	$\{*\}$ $\{/\}$



2 * 40 - 18 * 3 $



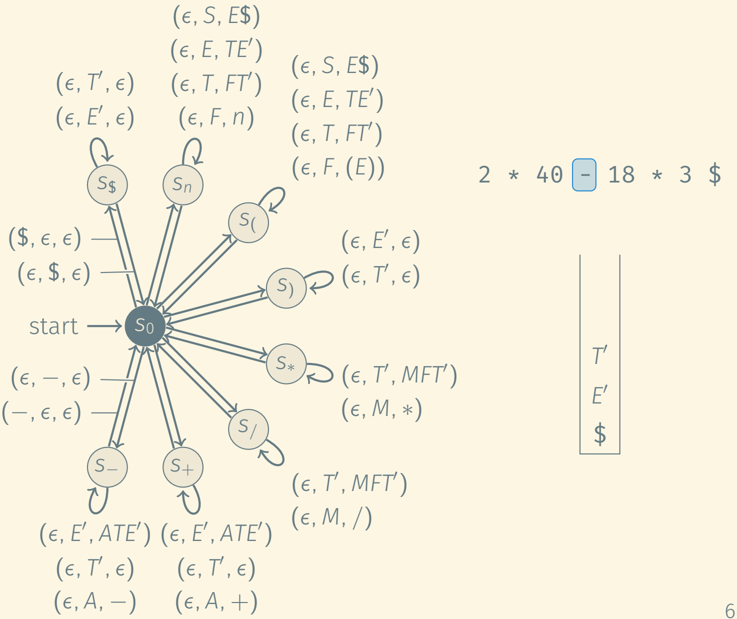
Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$



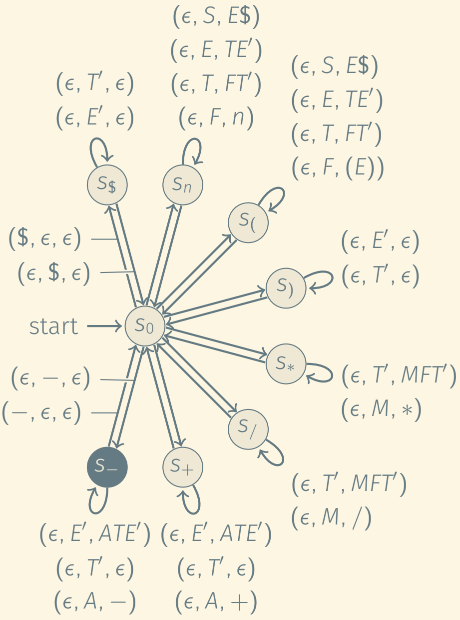
`2 * 40 - 18 * 3 $`

Stack
n
T'
E'
$\$$

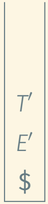
Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$



Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

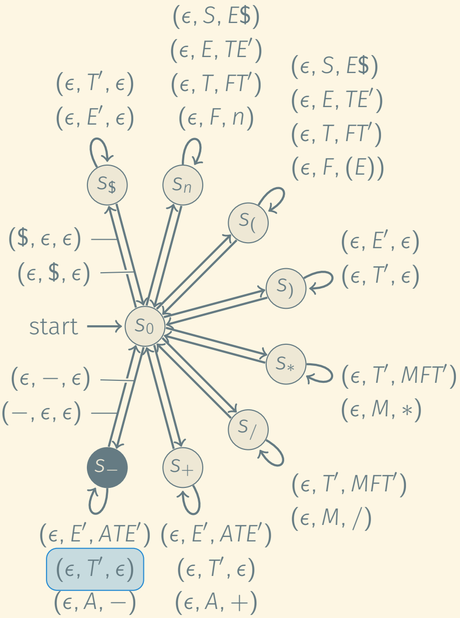


`2 * 40 - 18 * 3 $`



PARSING LL(1) LANGUAGES USING DPDA (1)

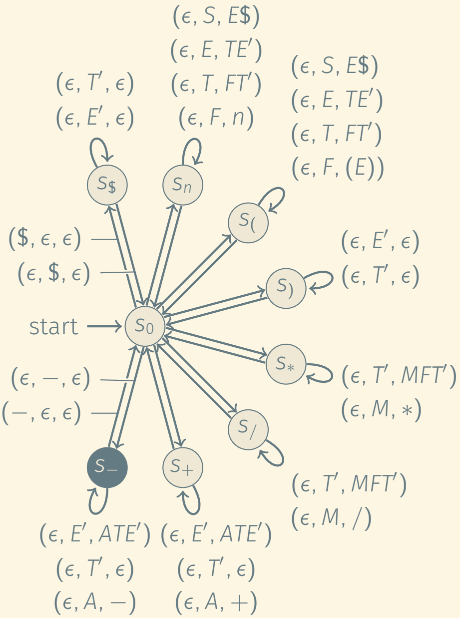
Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$



`2 * 40 - 18 * 3 $`

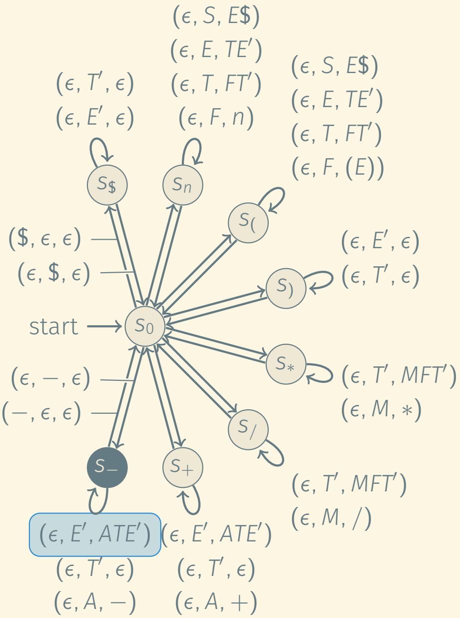
T'
E'
$\$$

Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$



`2 * 40 - 18 * 3 $`

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

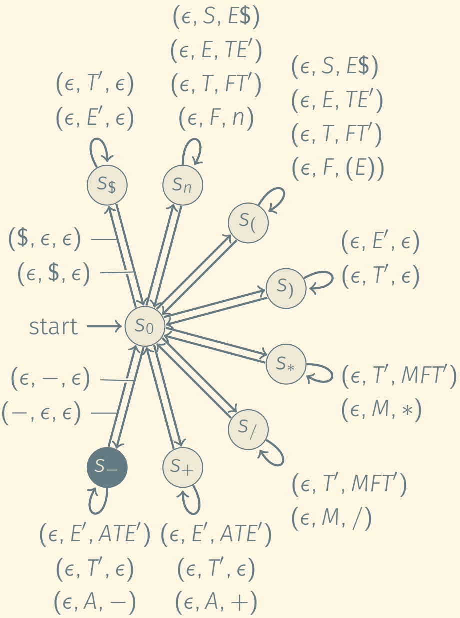


2 * 40 - 18 * 3 $

Stack: E', $\$$

PARSING LL(1) LANGUAGES USING DPDA (1)

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

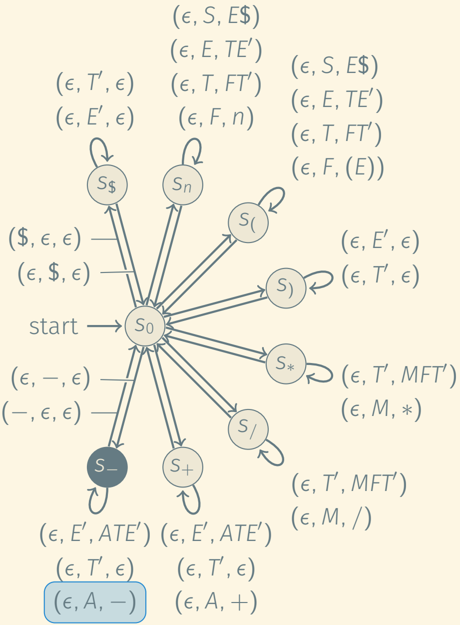


`2 * 40 - 18 * 3 $`

Stack
A
T
E'
$\$$

PARSING LL(1) LANGUAGES USING DPDA (1)

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$ $E' \rightarrow A\ T\ E'$	$\{\$,)\}$ $\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$ $T' \rightarrow M\ F\ T'$	$\{+, -, \$,)\}$ $\{*, /\}$
$F \rightarrow n$ $F \rightarrow (E)$	$\{n\}$ $\{(\}$
$A \rightarrow +$ $A \rightarrow -$	$\{+\}$ $\{-\}$
$M \rightarrow *$ $M \rightarrow /$	$\{*\}$ $\{/\}$

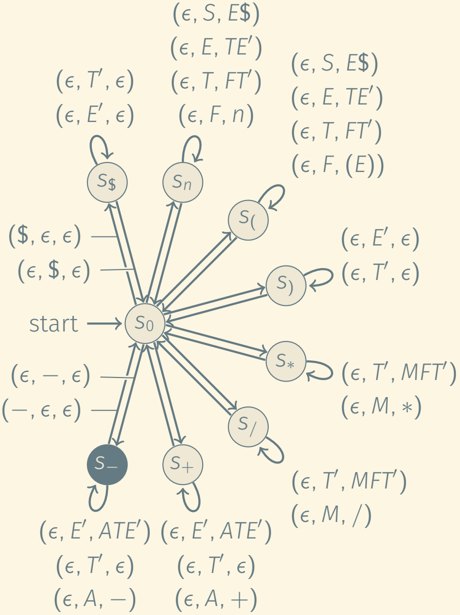


`2 * 40 - 18 * 3 $`

A
T
E'
$\$$

PARSING LL(1) LANGUAGES USING DPDA (1)

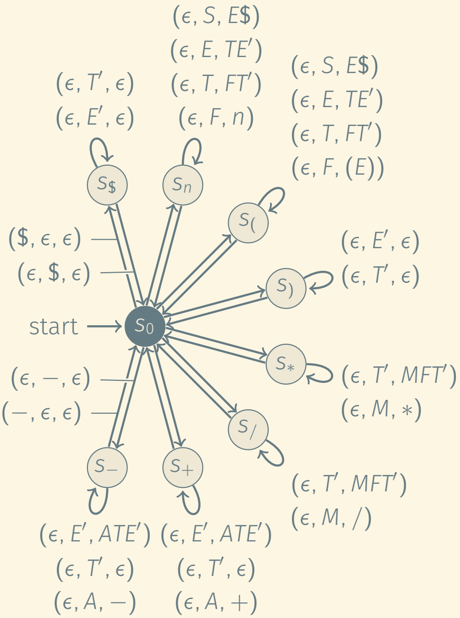
Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

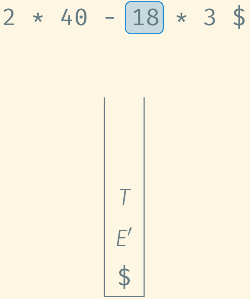


`2 * 40 - 18 * 3 $`

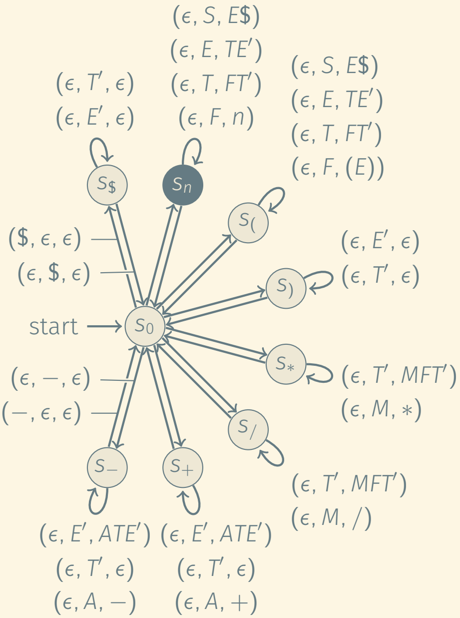
PARSING LL(1) LANGUAGES USING DPDA (1)

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$ $E' \rightarrow A\ T\ E'$	$\{\$,)\}$ $\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$ $T' \rightarrow M\ F\ T'$	$\{+, -, \$,)\}$ $\{*, /\}$
$F \rightarrow n$ $F \rightarrow (E)$	$\{n\}$ $\{(\}$
$A \rightarrow +$ $A \rightarrow -$	$\{+\}$ $\{-\}$
$M \rightarrow *$ $M \rightarrow /$	$\{*\}$ $\{/\}$





Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$



`2 * 40 - 18 * 3 $`

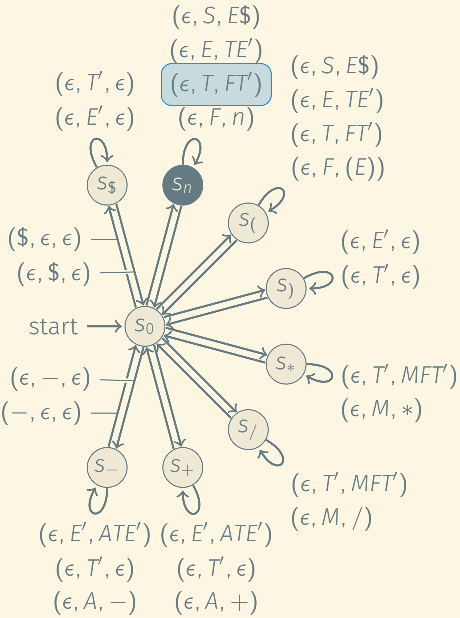
T

E'

$\$$

PARSING LL(1) LANGUAGES USING DPDA (1)

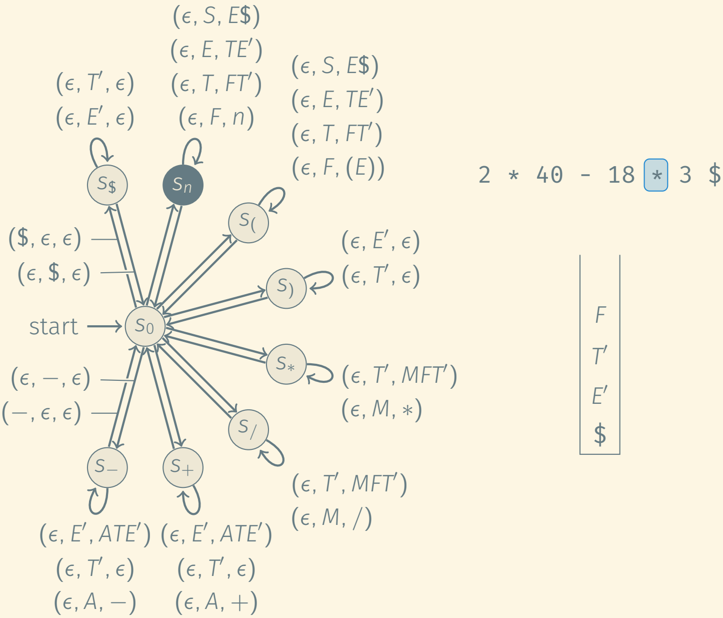
Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$ $E' \to A\ T\ E'$	$\{\$,)\}$ $\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$ $T' \to M\ F\ T'$	$\{+, -, \$,)\}$ $\{*, /\}$
$F \to n$ $F \to (E)$	$\{n\}$ $\{(\}$
$A \to +$ $A \to -$	$\{+\}$ $\{-\}$
$M \to *$ $M \to /$	$\{*\}$ $\{/\}$



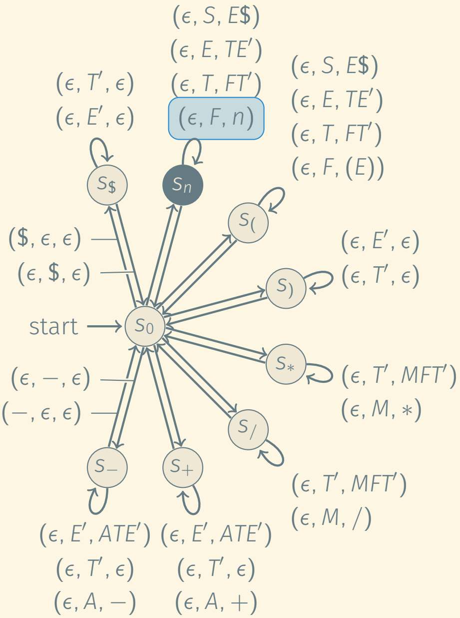
`2 * 40 - 18 * 3 $`

Stack
T
E'
$\$$

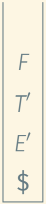
Rule R	PREDICT(R)
$S \to E\,\$$	$\{n, (\}$
$E \to T\,E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\,T\,E'$	$\{+, -\}$
$T \to F\,T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\,F\,T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$



Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

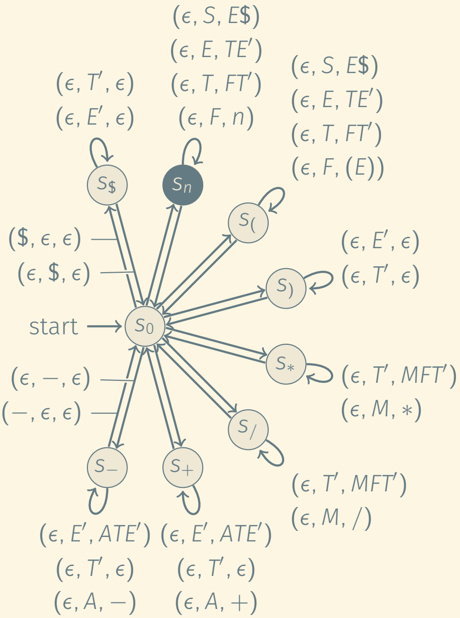


`2 * 40 - 18 * 3 $`



PARSING LL(1) LANGUAGES USING DPDA (1)

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

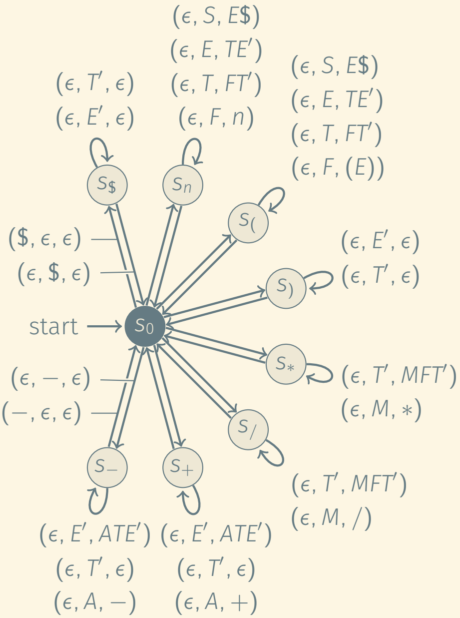


`2 * 40 - 18 * 3 $`

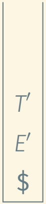
n
T'
E'
$\$$

PARSING LL(1) LANGUAGES USING DPDA (1)

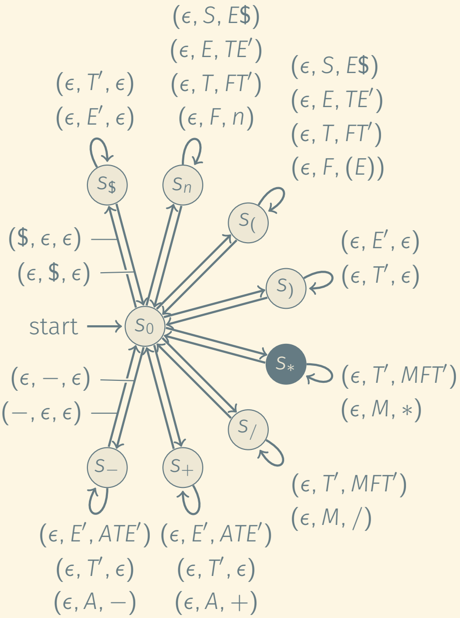
Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$



`2 * 40 - 18 * 3 $`



Rule R	PREDICT(R)
$S \rightarrow E\,\$$	$\{n, (\}$
$E \rightarrow T\,E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\,T\,E'$	$\{+, -\}$
$T \rightarrow F\,T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\,F\,T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

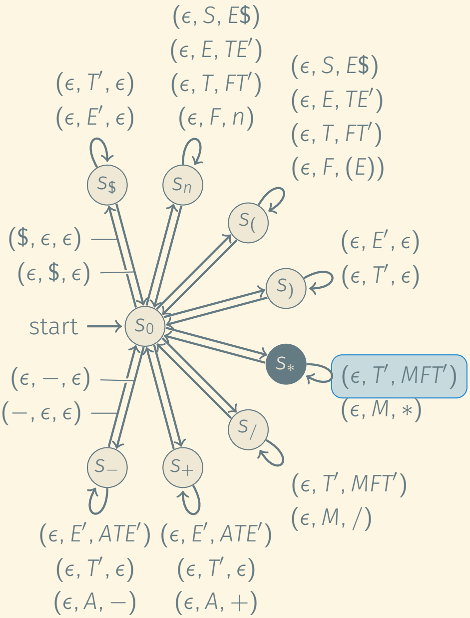


`2 * 40 - 18 * 3 $`

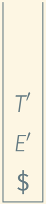
Stack:

T'
E'
$\$$

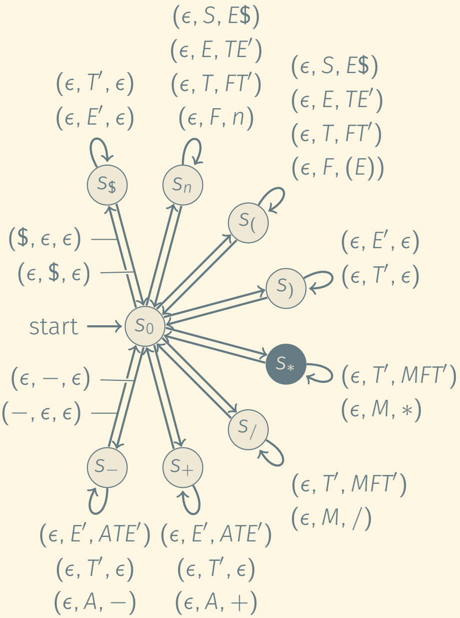
Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$



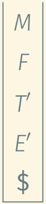
`2 * 40 - 18 * 3 $`



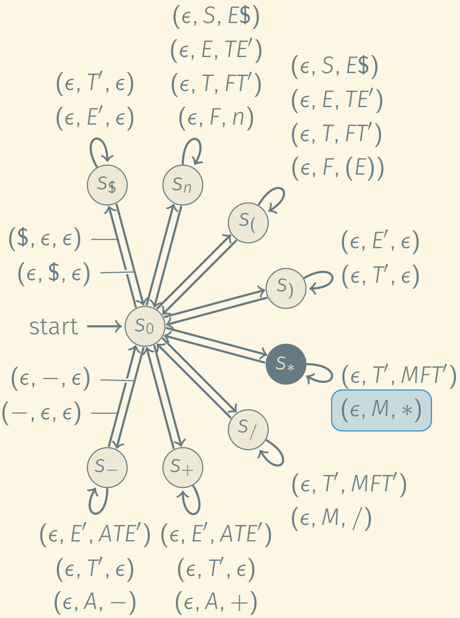
Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$



`2 * 40 - 18 * 3 $`



Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

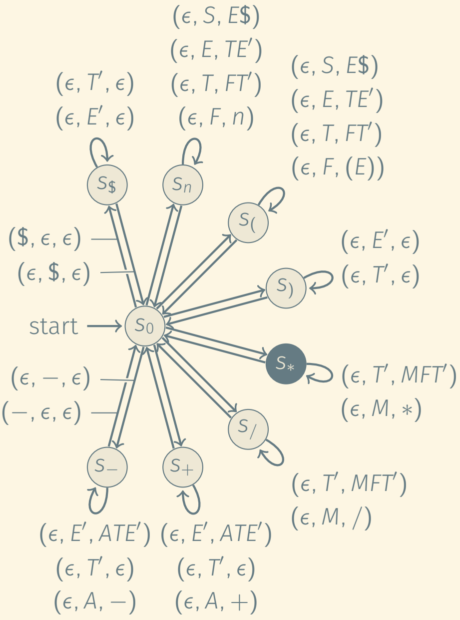


`2 * 40 - 18 * 3 $`

Stack
M
F
T'
E'
$\$$

PARSING LL(1) LANGUAGES USING DPDA (1)

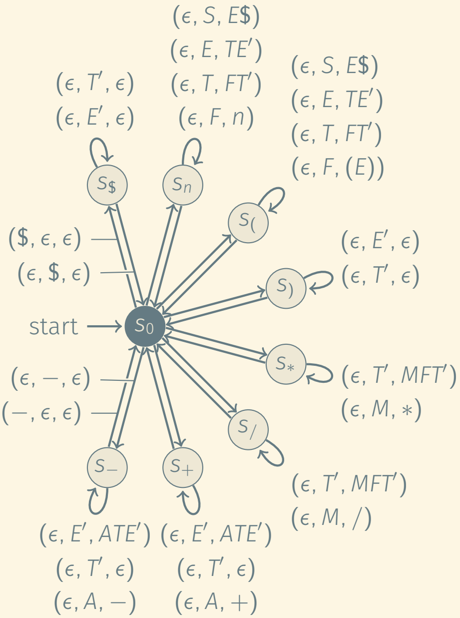
Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$



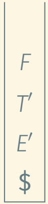
`2 * 40 - 18 * 3 $`

Stack
$*$
F
T'
E'
$\$$

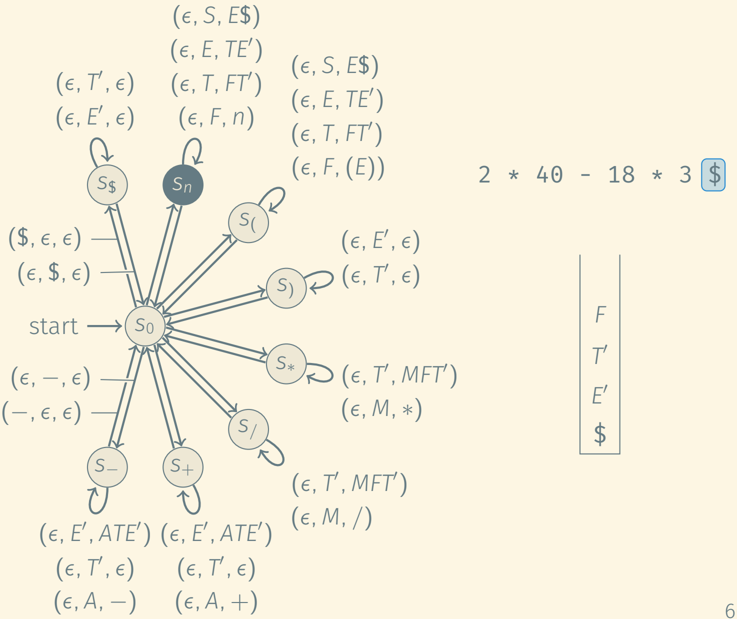
Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$



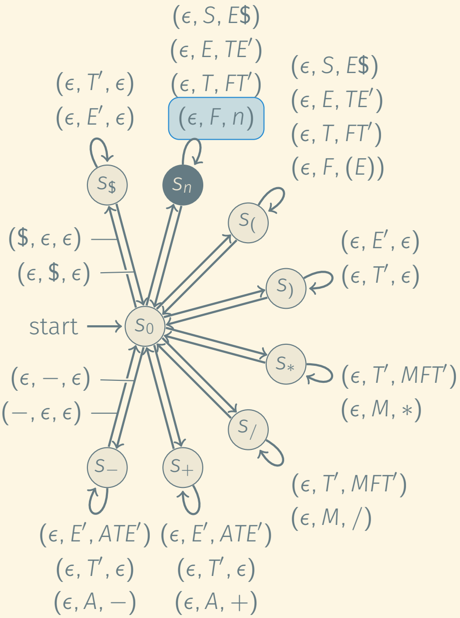
`2 * 40 - 18 * 3 $`



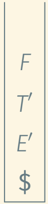
Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$ $E' \rightarrow A\ T\ E'$	$\{\$,)\}$ $\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$ $T' \rightarrow M\ F\ T'$	$\{+, -, \$,)\}$ $\{*, /\}$
$F \rightarrow n$ $F \rightarrow (E)$	$\{n\}$ $\{(\}$
$A \rightarrow +$ $A \rightarrow -$	$\{+\}$ $\{-\}$
$M \rightarrow *$ $M \rightarrow /$	$\{*\}$ $\{/\}$



Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

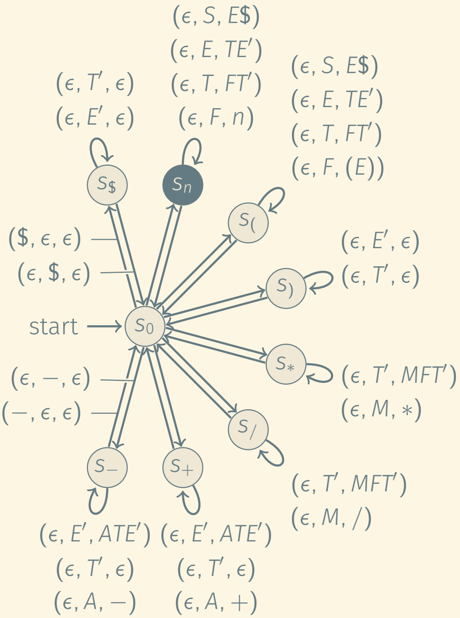


`2 * 40 - 18 * 3 $`



PARSING LL(1) LANGUAGES USING DPDA (1)

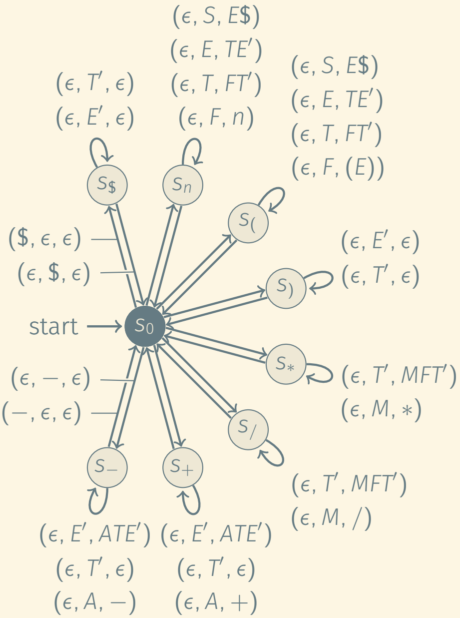
Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$



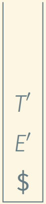
`2 * 40 - 18 * 3 $`

n
T'
E'
$\$$

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$

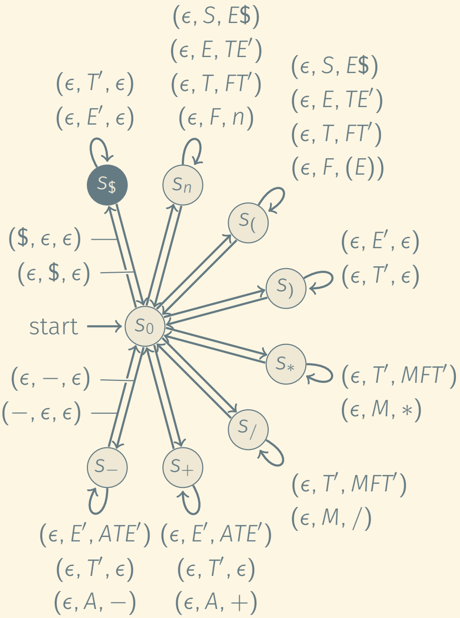


2 * 40 - 18 * 3 $

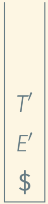


PARSING LL(1) LANGUAGES USING DPDA (1)

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$

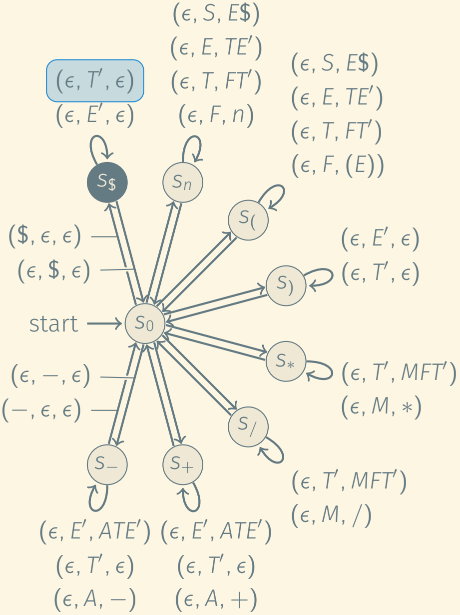


`2 * 40 - 18 * 3 $`



PARSING LL(1) LANGUAGES USING DPDA (1)

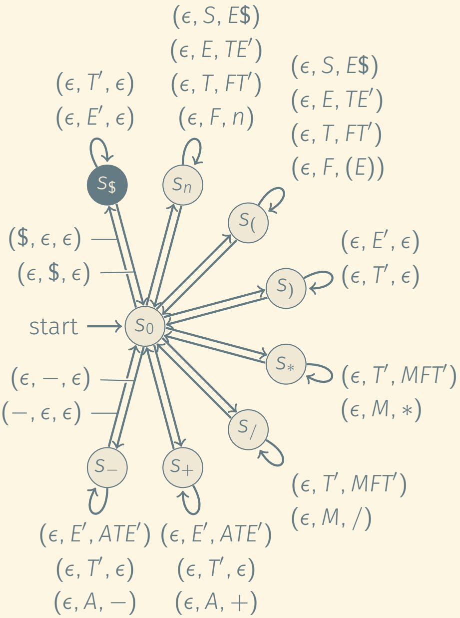
Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$ $E' \to A\ T\ E'$	$\{\$,)\}$ $\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$ $T' \to M\ F\ T'$	$\{+, -, \$,)\}$ $\{*, /\}$
$F \to n$ $F \to (E)$	$\{n\}$ $\{(\}$
$A \to +$ $A \to -$	$\{+\}$ $\{-\}$
$M \to *$ $M \to /$	$\{*\}$ $\{/\}$



`2 * 40 - 18 * 3 $`

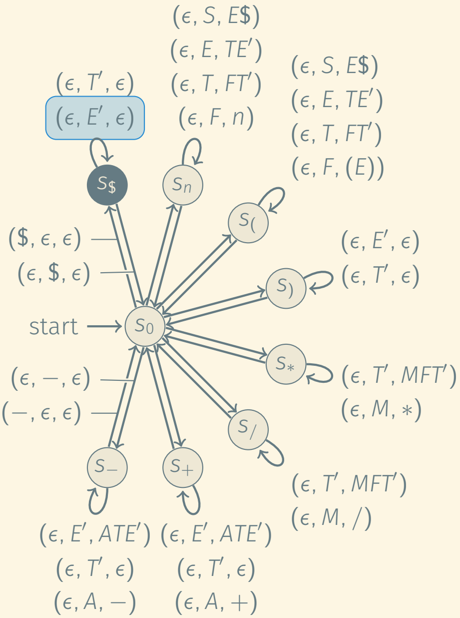
Stack
T'
E'
$\$$

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$ $E' \rightarrow A\ T\ E'$	$\{\$,)\}$ $\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$ $T' \rightarrow M\ F\ T'$	$\{+, -, \$,)\}$ $\{*, /\}$
$F \rightarrow n$ $F \rightarrow (E)$	$\{n\}$ $\{(\}$
$A \rightarrow +$ $A \rightarrow -$	$\{+\}$ $\{-\}$
$M \rightarrow *$ $M \rightarrow /$	$\{*\}$ $\{/\}$



`2 * 40 - 18 * 3 $`

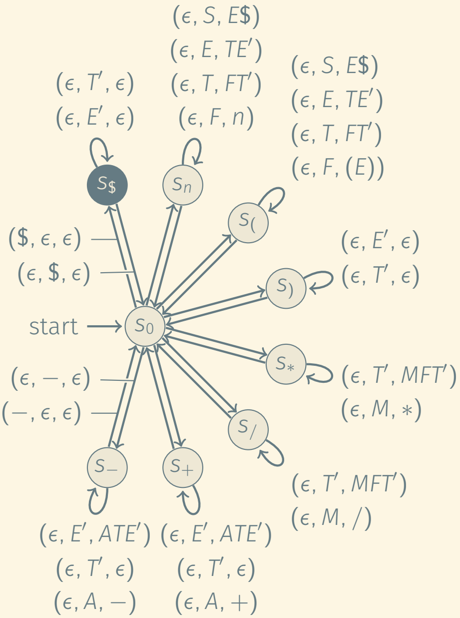
Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$



2 * 40 - 18 * 3 $

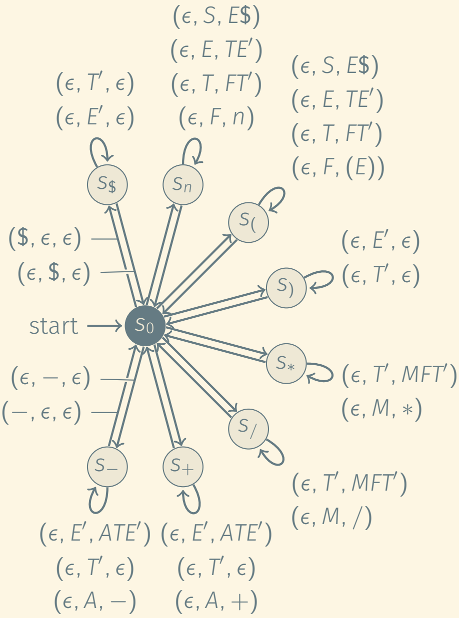
PARSING LL(1) LANGUAGES USING DPDA (1)

Rule R	PREDICT(R)
$S \to E\ \$$	$\{n, (\}$
$E \to T\ E'$	$\{n, (\}$
$E' \to \epsilon$	$\{\$,)\}$
$E' \to A\ T\ E'$	$\{+, -\}$
$T \to F\ T'$	$\{n, (\}$
$T' \to \epsilon$	$\{+, -, \$,)\}$
$T' \to M\ F\ T'$	$\{*, /\}$
$F \to n$	$\{n\}$
$F \to (E)$	$\{(\}$
$A \to +$	$\{+\}$
$A \to -$	$\{-\}$
$M \to *$	$\{*\}$
$M \to /$	$\{/\}$



2 * 40 - 18 * 3 $

Rule R	PREDICT(R)
$S \rightarrow E\ \$$	$\{n, (\}$
$E \rightarrow T\ E'$	$\{n, (\}$
$E' \rightarrow \epsilon$	$\{\$,)\}$
$E' \rightarrow A\ T\ E'$	$\{+, -\}$
$T \rightarrow F\ T'$	$\{n, (\}$
$T' \rightarrow \epsilon$	$\{+, -, \$,)\}$
$T' \rightarrow M\ F\ T'$	$\{*, /\}$
$F \rightarrow n$	$\{n\}$
$F \rightarrow (E)$	$\{(\}$
$A \rightarrow +$	$\{+\}$
$A \rightarrow -$	$\{-\}$
$M \rightarrow *$	$\{*\}$
$M \rightarrow /$	$\{/\}$



2 * 40 - 18 * 3 $

PARSING LL(1) LANGUAGES USING DPDA (2)

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Generating the parser:

- Hand-coded
- Automatic generation from grammar

SUMMARY

- Context-free grammars can be used to describe the structure of programming languages.
- Every context-free grammar can be parsed by PDA.
- Every context-free grammar can be parsed deterministically in $O(n^3)$ time.
- Linear-time parsing is possible for restricted grammars (S-grammar, LL(k), LR(k), ...).
- Tools: Recursive descent parser, shift-reduce parser, DPDA.