CSCI 3163 - Assignment 2 - Sample Solution

(a) This language is not regular. The proof is very similar to the one given for the language $\{0^n 1^n \mid n \geq 0\}$ given in class.

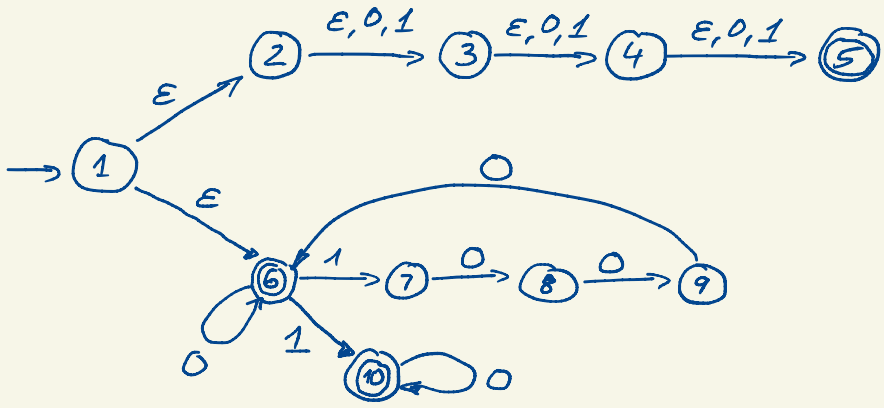
Assume it is regular. Then, by the Pumping Lemma, there exists an integer $n_{\mathcal{L}}$ such that any string $\sigma \in \mathcal{L}$ with $\sigma \geq n_{\mathcal{L}}$ can be written as the concatenation of three strings α, β, γ — $\sigma = \alpha\beta\gamma$ — such that $|\alpha\beta| \leq n_{\mathcal{L}}$, $|\beta| > 0$, and any string $\alpha\beta^k\gamma$ for $k \geq 0$ is also in $\mathcal{L}$.

Consider the string $0^{n_{\mathcal{L}}} 1^{n_{\mathcal{L}}} \in \mathcal{L}$. Since $|\alpha\beta| \leq n_{\mathcal{L}}$, we have $\alpha = 0^a$ and $\beta = 0^b$, where $b > 0$. By the Pumping Lemma, the string $\alpha\beta\beta\gamma = 0^{n_{\mathcal{L}}+b} 1^{n_{\mathcal{L}}}$ is also in $\mathcal{L}$, a contradiction because $n_{\mathcal{L}} + b \neq n_{\mathcal{L}}$, that is, $\alpha\beta\beta\gamma$ does not have the same number of 0s and 1s.

(b) This language is regular. A regular expression that describes it is

$$(0|1|\varepsilon)(0|1|\varepsilon)(0|1|\varepsilon) \mid (0|1000)^*(\varepsilon|10^*)$$

A corresponding NFA is



Conversion to DFA:

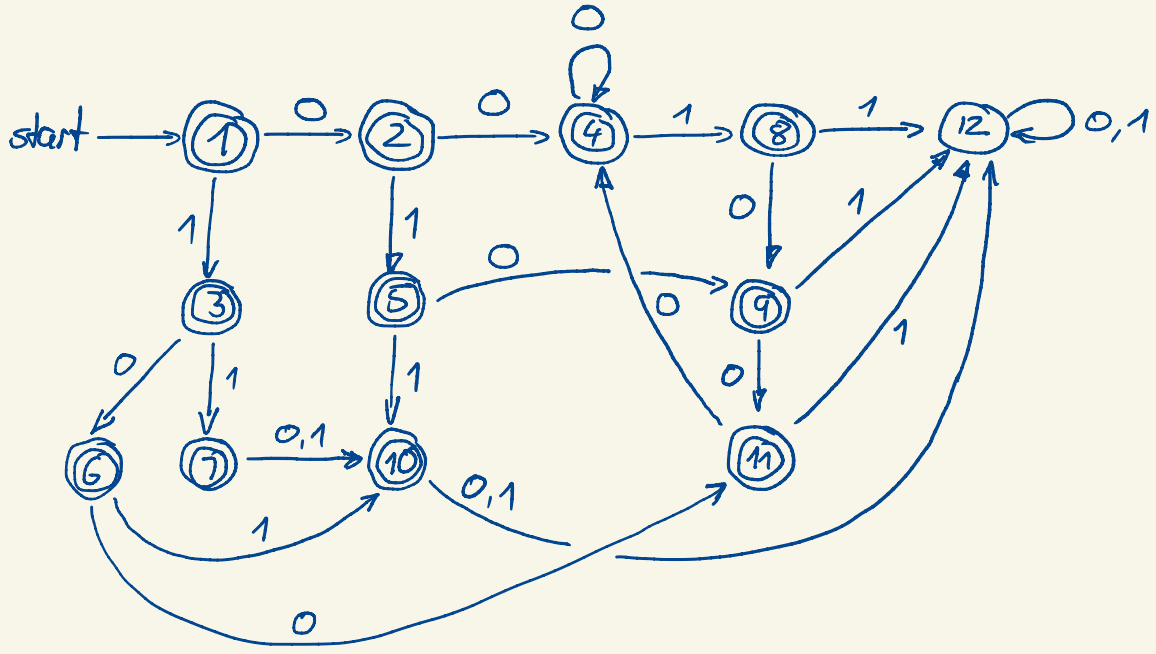
	DFA State	0	1	State number
start →	* {1,2,3,4,5,6}	{3,4,5,6}	{3,4,5,7,10}	1
	* {3,4,5,6}	{4,5,6}	{4,5,7,10}	2
	* {3,4,5,7,10}	{4,5,8,10}	{4,5}	3
	* {4,5,6}	{5,6}	{5,7,10}	4
	* {4,5,7,10}	{5,8,10}	{5}	5
	* {4,5,8,10}	{5,9,10}	{5}	6
	* {4,5}	{5}	{5}	7
	* {5,6}	{6}	{7,10}	8
	* {5,7,10}	{8,10}	∅	9
	* {5,8,10}	{9,10}	∅	10
	* {5}	∅	∅	11
	* {5,9,10}	{6,10}	∅	12
	* {6}	{6}	{7,10}	13
	* {7,10}	{8,10}	∅	14
	* {8,10}	{9,10}	∅	15
	∅	∅	∅	16
	* {9,10}	{6,10}	∅	17
	* {6,10}	{6,10}	{7,10}	18

This gives the following table with each state represented by the numbes in the rightmost column above

Equivalence classes

	State	0	1	H.1 Class	Target class 0	Target class 1	H.2. Class	Target class 0	Target class 1	H.3 Class	Target class 0	Target class 1	H.4 Class	Target class 0	Target class 1	Final class
→	* 1	2	3	1	1	1	1	1	1	1	1	1	1	2	3	1
	* 2	4	5	1	1	1	1	1	1	1	2	3	2	4	5	2
	* 3	6	7	1	1	1	1	1	1	1	3	4	3	6	7	3
	* 4	8	9	1	1	1	1	1	2	2	2	5	4	4	8	4
	* 5	10	11	1	1	1	1	2	3	3	5	6	5	9	10	5
	* 6	12	11	1	1	1	1	2	3	3	7	6	6	11	10	6
	* 7	11	11	1	1	1	1	3	3	4	6	6	7	10	10	7
	* 8	13	14	1	1	1	1	1	2	2	2	5	4	4	8	4
	* 9	15	16	1	1	2	2	2	4	5	5	8	8	9	12	8
	* 10	17	16	1	1	2	2	2	4	5	7	8	9	11	12	9
	* 11	16	16	1	2	2	3	4	4	6	8	8	10	12	12	10
	* 12	18	16	1	1	2	2	1	4	7	2	8	11	4	12	11
	* 13	13	14	1	1	1	1	1	2	2	2	5	4	4	8	4
	* 14	15	16	1	1	2	2	2	4	5	5	8	8	9	12	8
	* 15	17	16	1	1	2	2	2	4	5	7	8	9	11	12	9
	16	16	16	2	2	2	4	4	4	8	8	8	12	12	12	12
	* 17	18	16	1	1	2	2	1	4	7	2	8	11	4	12	11
	* 18	18	14	1	1	1	1	1	2	2	2	5	4	4	8	4

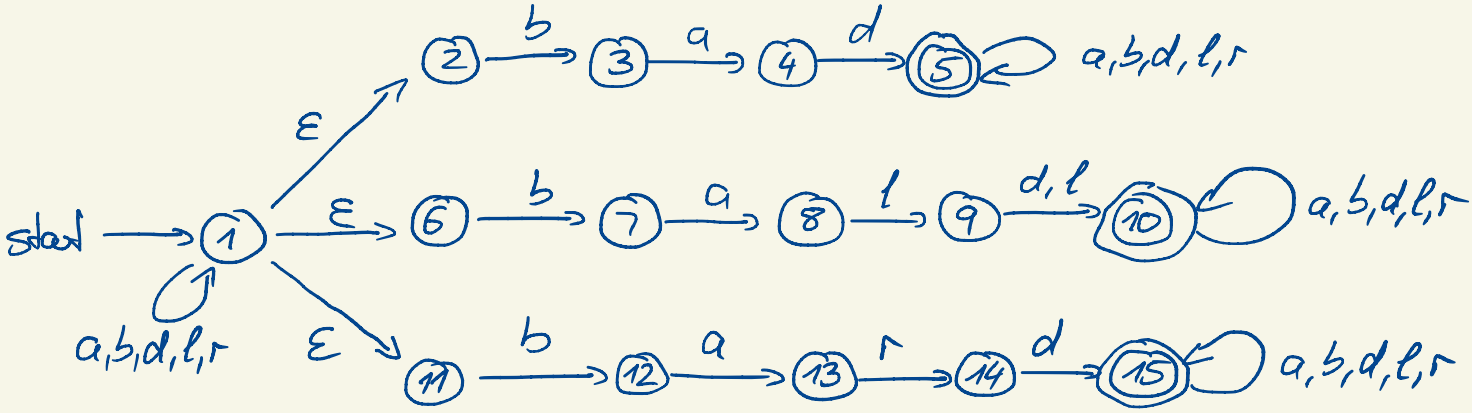
The corresponding DFA looks as follows:



(c) This language is regulas. The regulas expression is

(a|b|d|ℓ|r)*(bad|bald|ball|bard)(a|b|d|ℓ|r)*

An NFA that accepts this language is



The convesion to a DFA is on the next page:

	State	a	b	d	ℓ	r	#
→	{1,2,6,11}	{1,2,6,11}	{1,2,6,11,3,7,12}	{1,2,6,11}	{1,2,6,11}	{1,2,6,11}	1
	{1,2,6,11,3,7,12}	{1,2,6,11,4,8,13}	{1,2,6,11,3,7,12}	{1,2,6,11}	{1,2,6,11}	{1,2,6,11}	2
	{1,2,6,11,4,8,13}	{1,2,6,11}	{1,2,6,11,3,7,12}	{1,2,6,11,5}	{1,2,6,11,9}	{1,2,6,11,14}	3
*	{1,2,6,11,5}	{1,2,6,11,5}	{1,2,6,11,3,7,12,5}	{1,2,6,11,5}	{1,2,6,11,5}	{1,2,6,11,5}	4
	{1,2,6,11,9}	{1,2,6,11}	{1,2,6,11,3,7,12}	{1,2,6,11,10}	{1,2,6,11,10}	{1,2,6,11}	5
	{1,2,6,11,14}	{1,2,6,11}	{1,2,6,11,3,7,12}	{1,2,6,11,15}	{1,2,6,11}	{1,2,6,11}	6
*	{1,2,6,11,3,7,12,5}	{1,2,6,11,4,8,13,5}	{1,2,6,11,3,7,12,5}	{1,2,6,11,5}	{1,2,6,11,5}	{1,2,6,11,5}	7
*	{1,2,6,11,10}	{1,2,6,11,10}	{1,2,6,11,3,7,12,10}	{1,2,6,11,10}	{1,2,6,11,10}	{1,2,6,11,10}	8
*	{1,2,6,11,15}	{1,2,6,11,15}	{1,2,6,11,3,7,12,15}	{1,2,6,11,15}	{1,2,6,11,15}	{1,2,6,11,15}	9
*	{1,2,6,11,4,8,13,5}	{1,2,6,11,5}	{1,2,6,11,3,7,12,5}	{1,2,6,11,5}	{1,2,6,11,9,5}	{1,2,6,11,14,5}	10
*	{1,2,6,11,3,7,12,10}	{1,2,6,11,4,8,13,10}	{1,2,6,11,3,7,12,10}	{1,2,6,11,10}	{1,2,6,11,10}	{1,2,6,11,10}	11
*	{1,2,6,11,3,7,12,15}	{1,2,6,11,4,8,13,15}	{1,2,6,11,3,7,12,15}	{1,2,6,11,15}	{1,2,6,11,15}	{1,2,6,11,15}	12
*	{1,2,6,11,9,5}	{1,2,6,11,5}	{1,2,6,11,3,7,12,5}	{1,2,6,11,10,5}	{1,2,6,11,10,5}	{1,2,6,11,5}	13
*	{1,2,6,11,14,5}	{1,2,6,11,5}	{1,2,6,11,3,7,12,5}	{1,2,6,11,15,5}	{1,2,6,11,5}	{1,2,6,11,5}	14
*	{1,2,6,11,4,8,13,10}	{1,2,6,11,10}	{1,2,6,11,3,7,12,10}	{1,2,6,11,5,10}	{1,2,6,11,9,10}	{1,2,6,11,14,10}	15
*	{1,2,6,11,4,8,13,15}	{1,2,6,11,15}	{1,2,6,11,3,7,12,15}	{1,2,6,11,5,15}	{1,2,6,11,9,15}	{1,2,6,11,14,15}	16
*	{1,2,6,11,10,5}	{1,2,6,11,10,5}	{1,2,6,11,3,7,12,10,5}	{1,2,6,11,10,5}	{1,2,6,11,10,5}	{1,2,6,11,10,5}	17
*	{1,2,6,11,15,5}	{1,2,6,11,15,5}	{1,2,6,11,3,7,12,15,5}	{1,2,6,11,15,5}	{1,2,6,11,15,5}	{1,2,6,11,15,5}	18
*	{1,2,6,11,9,10}	{1,2,6,11,10}	{1,2,6,11,3,7,12,10}	{1,2,6,11,10}	{1,2,6,11,10}	{1,2,6,11,10}	19
*	{1,2,6,11,14,10}	{1,2,6,11,10}	{1,2,6,11,3,7,12,10}	{1,2,6,11,15,10}	{1,2,6,11,10}	{1,2,6,11,10}	20

*	{1,2,6,11,9,15}	{1,2,6,11,15}	{1,2,6,11,3,7,12,15}	{1,2,6,11,10,15}	{1,2,6,11,10,15}	{1,2,6,11,15}	21
*	{1,2,6,11,14,15}	{1,2,6,11,15}	{1,2,6,11,3,7,12,15}	{1,2,6,11,15}	{1,2,6,11,15}	{1,2,6,11,15}	22
*	{1,2,6,11,3,7,12,10,5}	{1,2,6,11,4,8,13,10,5}	{1,2,6,11,3,7,12,10,5}	{1,2,6,11,10,5}	{1,2,6,11,10,5}	{1,2,6,11,10,5}	23
*	{1,2,6,11,3,7,12,15,5}	{1,2,6,11,4,8,13,15,5}	{1,2,6,11,3,7,12,15,5}	{1,2,6,11,15,5}	{1,2,6,11,15,5}	{1,2,6,11,15,5}	24
*	{1,2,6,11,15,10}	{1,2,6,11,15,10}	{1,2,6,11,3,7,12,15,10}	{1,2,6,11,15,10}	{1,2,6,11,15,10}	{1,2,6,11,15,10}	25
*	{1,2,6,11,4,8,13,10,5}	{1,2,6,11,10,5}	{1,2,6,11,3,7,12,10,5}	{1,2,6,11,10,5}	{1,2,6,11,9,10,5}	{1,2,6,11,14,10,5}	26
*	{1,2,6,11,4,8,13,15,5}	{1,2,6,11,15,5}	{1,2,6,11,3,7,12,15,5}	{1,2,6,11,15,5}	{1,2,6,11,9,15,5}	{1,2,6,11,14,15,5}	27
*	{1,2,6,11,3,7,12,15,10}	{1,2,6,11,4,8,13,15,10}	{1,2,6,11,3,7,12,15,10}	{1,2,6,11,15,10}	{1,2,6,11,15,10}	{1,2,6,11,15,10}	28
*	{1,2,6,11,9,10,5}	{1,2,6,11,10,5}	{1,2,6,11,3,7,12,10,5}	{1,2,6,11,10,5}	{1,2,6,11,10,5}	{1,2,6,11,10,5}	29
*	{1,2,6,11,14,10,5}	{1,2,6,11,10,5}	{1,2,6,11,3,7,12,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,10,5}	{1,2,6,11,10,5}	30
*	{1,2,6,11,9,15,5}	{1,2,6,11,15,5}	{1,2,6,11,3,7,12,15,5}	{1,2,6,11,10,15,5}	{1,2,6,11,10,15,5}	{1,2,6,11,15,5}	31
*	{1,2,6,11,14,15,5}	{1,2,6,11,15,5}	{1,2,6,11,3,7,12,15,5}	{1,2,6,11,15,5}	{1,2,6,11,15,5}	{1,2,6,11,15,5}	32
*	{1,2,6,11,4,8,13,15,10}	{1,2,6,11,15,10}	{1,2,6,11,3,7,12,15,10}	{1,2,6,11,5,15,10}	{1,2,6,11,9,15,10}	{1,2,6,11,14,15,10}	33
*	{1,2,6,11,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,3,7,12,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,15,10,5}	34
*	{1,2,6,11,9,15,10}	{1,2,6,11,15,10}	{1,2,6,11,3,7,12,15,10}	{1,2,6,11,15,10}	{1,2,6,11,15,10}	{1,2,6,11,15,10}	35
*	{1,2,6,11,14,15,10}	{1,2,6,11,15,10}	{1,2,6,11,3,7,12,15,10}	{1,2,6,11,15,10}	{1,2,6,11,15,10}	{1,2,6,11,15,10}	36
*	{1,2,6,11,3,7,12,15,10,5}	{1,2,6,11,4,8,13,15,10,5}	{1,2,6,11,3,7,12,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,15,10,5}	37
*	{1,2,6,11,4,8,13,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,3,7,12,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,9,15,10,5}	{1,2,6,11,14,15,10,5}	38
*	{1,2,6,11,9,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,3,7,12,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,15,10,5}	39
*	{1,2,6,11,14,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,3,7,12,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,15,10,5}	{1,2,6,11,15,10,5}	40

By numbering the states, we obtain the following transition table:

Equivalence classes

State	a	b	d	l	r	H.1 class	Target classes a b d l r	H.2 class	Target class a b d l r	H.3 class	Target class a b d l r	Final class
→ 1	1	2	1	1	1	1	1 1 1 1 1	1	1 1 1 1 1	1	1 2 1 1 1	1
2	3	2	1	1	1	1	1 1 1 1 1	1	2 1 1 1 1	2	3 2 1 1 1	2
3	1	2	4	5	6	1	1 1 2 1 1	2	1 1 3 4 2	3	1 2 4 5 6	3
* 4	4	7	4	4	4	2	2 2 2 2 2	3	2 2 2 2 2	4	4 4 4 4 4	4
5	1	2	8	8	1	1	1 1 2 2 1	4	1 1 3 3 1	5	1 2 4 4 1	5
6	1	2	9	1	1	1	1 1 2 1 1	2	1 1 3 1 1	6	1 2 4 1 1	6
* 7	10	7	4	4	4	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 8	8	11	8	8	8	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 9	9	12	9	9	9	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 10	4	7	4	13	14	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 11	15	11	8	8	8	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 12	16	12	9	9	9	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 13	4	7	17	17	4	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 14	4	7	18	4	4	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 15	8	11	17	19	20	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 16	9	12	18	21	22	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 17	17	23	17	17	17	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 18	18	24	18	18	18	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 19	8	11	8	8	8	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 20	8	11	25	8	8	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 21	9	12	25	25	9	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 22	9	12	9	9	9	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 23	26	23	17	17	17	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 24	27	24	18	18	18	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 25	25	28	25	25	25	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 26	17	23	17	29	30	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 27	18	24	18	31	32	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 28	33	28	25	25	25	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 29	17	23	17	17	17	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 30	17	23	34	17	17	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 31	18	24	34	34	18	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 32	18	24	18	18	18	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 33	25	28	34	35	36	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 34	34	37	34	34	34	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 35	25	28	25	25	25	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 36	25	28	25	25	25	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 37	38	37	34	34	34	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 38	34	37	34	39	40	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 39	34	37	34	34	34	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4
* 40	34	37	34	34	34	2	2 2 2 2 2	3	3 3 3 3 3	4	4 4 4 4 4	4

The corresponding DFA is: