

**Assignment 3**  
**CSCI 3110: Design and Analysis of Algorithms**  
Due Jun 5, 2018

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Assignments are due on the due date before class and have to include this cover page. Plagiarism in assignment answers will not be tolerated. By submitting their answers to this assignment, the authors named above declare that its content is their original work and that they did not use any sources for its preparation other than the class notes, the textbook, and ones explicitly acknowledged in the answers. Any suspected act of plagiarism will be reported to the Faculty's Academic Integrity Officer and possibly to the Senate Discipline Committee. The penalty for academic dishonesty may range from failing the course to expulsion from the university, in accordance with Dalhousie University's regulations regarding academic integrity.

Here's a beautiful question I found in an algorithms textbook. It has a very simple solution, but you have to be a little bit creative:

You're helping a group of ethnographers analyze some oral history data they've collected by interviewing members of a village to learn about the lives of people who've lived there over the past two hundred years.

From these interviews, they've learned about a set of  $n$  people (all of them now deceased), whom we'll denote  $P_1, P_2, \dots, P_n$ . They've also collected facts about when these people lived relative to one another. Each fact has one of the following two forms:

- For some  $i$  and  $j$ , person  $P_i$  died before person  $P_j$  was born; or
- For some  $i$  and  $j$ , the life spans of  $P_i$  and  $P_j$  overlapped at least partially.

Naturally, they're not sure that all these facts are correct; memories are not so good, and a lot of this was passed down by word of mouth. So what they'd like you to determine is whether the data they've collected is at least internally consistent, in the sense that there could have existed a set of people for which all the facts they've learned hold simultaneously.

Give an efficient algorithm to do this: either it should produce proposed dates of birth and death for each of the  $n$  people so that all the facts hold true, or it should report (correctly) that no such dates can exist—that is, the facts collected by the ethnographers are not internally consistent.

To make the question a bit more precise, you are asked to develop an algorithm that runs in  $O(n + m \log n)$  time, where  $n$  denotes the number of people and  $m$  denotes the number of facts collected about these people. Argue that your algorithm is correct and that its running time is indeed  $O(n + m \log n)$ .